Chapter 46: Distribution Functions

R has many built-in functions to work with probability distributions, with official docs starting at `?Distributions`.

Section 46.1: Normal distribution

Let's use `*norm` as an example. From the documentation:

```
dnorm(x, mean = 0, sd = 1, log = FALSE)
pnorm(q, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)
qnorm(p, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)
rnorm(n, mean = 0, sd = 1)
```

So if I wanted to know the value of a standard normal distribution at 0, I would do

```
dnorm(0)
```

Which gives us `0.3989423`, a reasonable answer.

In the same way `pnorm(0)` gives `.5`. Again, this makes sense, because half of the distribution is to the left of 0.

`qnorm` will essentially do the opposite of `pnorm`. `qnorm(.5)` gives `0`.

Finally, there's the `rnorm` function:

```
rnorm(10)
```

Will generate 10 samples from standard normal.

If you want to change the parameters of a given distribution, simply change them like so

`rnorm(10, mean=4, sd= 3)`

Section 46.2: Binomial Distribution

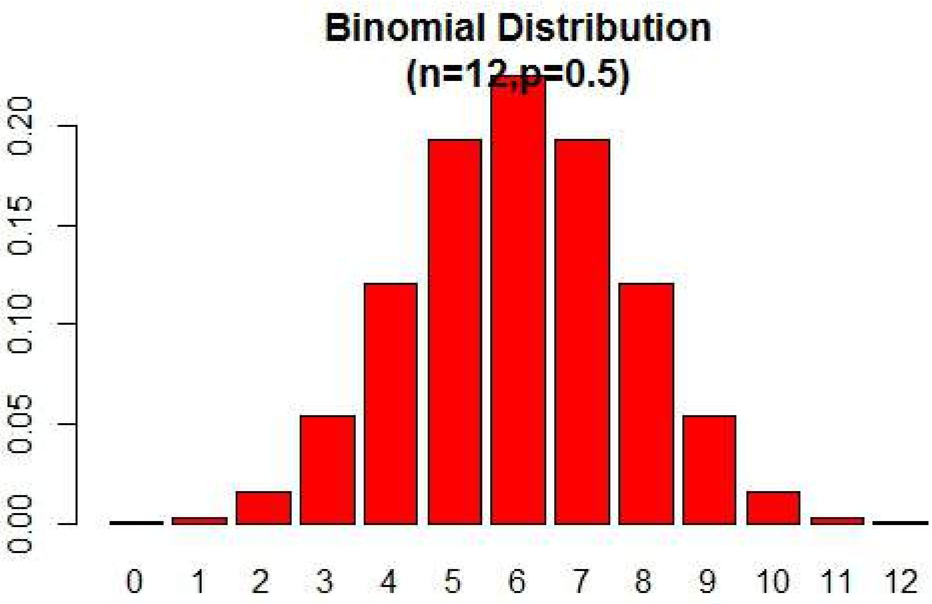
We now illustrate the functions `dbinom`,`pbinom`,`qbinom` and `rbinom` defined for *Binomial distribution*.

The `dbinom()` function gives the probabilities for various values of the binomial variable. Minimally it requires three arguments. The first argument for this function must be a vector of quantiles(the possible values of the random variable `X`). The second and third arguments are the `defining parameters` of the distribution, namely, `n`(the number of independent trials) and `p`(the probability of success in each trial). For example, for a binomial distribution with `n = 5`, `p = 0.5`, the possible values for X are `0`,`1`,`2`,`3`,`4`,`5`. That is, the `dbinom(x,n,p)` function gives the probability values `P( X = x )` for `x = 0, 1, 2, 3, 4, 5`.

```
#Binom(n = 5, p = 0.5) probabilities
> n <- 5; p<- 0.5; x <- 0:n
> dbinom(x,n,p)
[1] 0.03125 0.15625 0.31250 0.31250 0.15625 0.03125
#To verify the total probability is 1
> sum(dbinom(x,n,p))
[1] 1
>
```

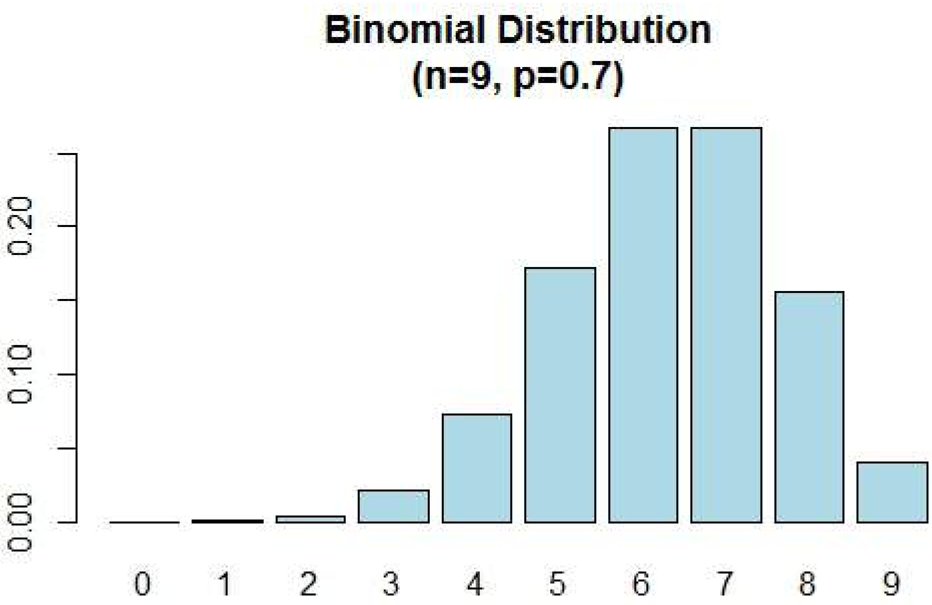
The binomial probability distribution plot can be displayed as in the following figure:

```
> x <- 0:12
> prob <- dbinom(x,12,.5)
> barplot(prob,col = "red",ylim = c(0,.2),names.arg=x,
                            main="Binomial Distribution\n(n=12,p=0.5)")
```



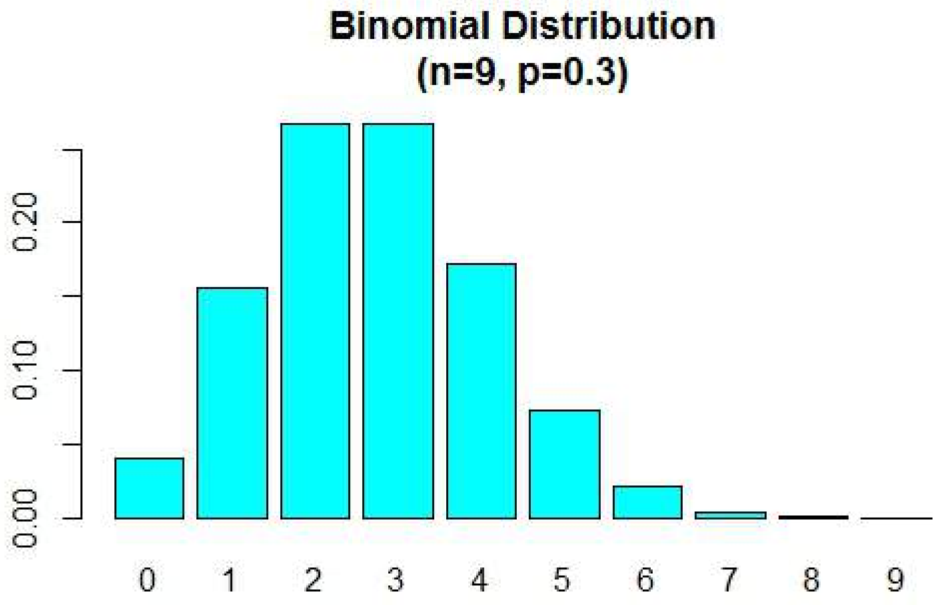
Note that the binomial distribution is symmetric when `p = 0.5`. To demonstrate that the binomial distribution is negatively skewed when `p` is larger than `0.5`, consider the following example:

```
> n=9; p=.7; x=0:n; prob=dbinom(x,n,p);
> barplot(prob,names.arg = x,main="Binomial Distribution\n(n=9, p=0.7)",col="lightblue")
```



When `p` is smaller than `0.5` the binomial distribution is positively skewed as shown below.

```
> n=9; p=.3; x=0:n; prob=dbinom(x,n,p);
> barplot(prob,names.arg = x,main="Binomial Distribution\n(n=9, p=0.3)",col="cyan")
```



We will now illustrate the usage of the cumulative distribution function `pbinom()`. This function can be used to calculate probabilities such as `P( X <= x )`. The first argument to this function is a vector of quantiles(values of x).

```
# Calculating Probabilities
# P(X <= 2) in a Bin(n=5,p=0.5) distribution
> pbinom(2,5,0.5)
[1] 0.5
```

The above probability can also be obtained as follows:

```
# P(X <= 2) = P(X=0) + P(X=1) + P(X=2)
> sum(dbinom(0:2,5,0.5))
[1] 0.5
```

To compute, probabilities of the type: `P( a <= X <= b )`

```
# P(3<= X <= 5) = P(X=3) + P(X=4) + P(X=5) in a Bin(n=9,p=0.6) dist
> sum(dbinom(c(3,4,5),9,0.6))
[1] 0.4923556
>
```

Presenting the binomial distribution in the form of a table:

```
> n = 10; p = 0.4; x = 0:n;
> prob = dbinom(x,n,p)
> cdf = pbinom(x,n,p)
> distTable = cbind(x,prob,cdf)
> distTable
      x         prob         cdf
 [1,] 0 0.0060466176 0.006046618
 [2,] 1 0.0403107840 0.046357402
 [3,] 2 0.1209323520 0.167289754
```

```
 [4,]  3 0.2149908480 0.382280602
 [5,]  4 0.2508226560 0.633103258
 [6,]  5 0.2006581248 0.833761382
 [7,]  6 0.1114767360 0.945238118
 [8,]  7 0.0424673280 0.987705446
 [9,]  8 0.0106168320 0.998322278
[10,]  9 0.0015728640 0.999895142
[11,] 10 0.0001048576 1.000000000
>
```

The `rbinom()` is used to generate random samples of specified sizes with a given parameter values.

```
# Simulation
> xVal<-names(table(rbinom(1000,8,.5)))
> barplot(as.vector(table(rbinom(1000,8,.5))),names.arg =xVal,
                   main="Simulated Binomial Distribution\n (n=8,p=0.5)")
```