



# Helping With Math

## Graphing and Solving Systems of Linear Equations in Two Variables

GRADE 8



Solving a system of linear equations means finding numerical values for all unknown variables that will make the equations of the system true. We can use graphing method, substitution and elimination method in finding the solution set of the systems of equations.



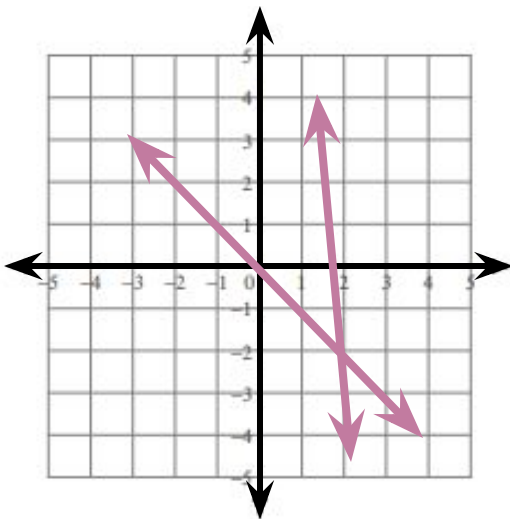
Hello! Come and enjoy the music festival after you study!

A **system of linear equations** is a set of two or more linear equations made up of two or more variables such that all equations in the system are considered simultaneously.

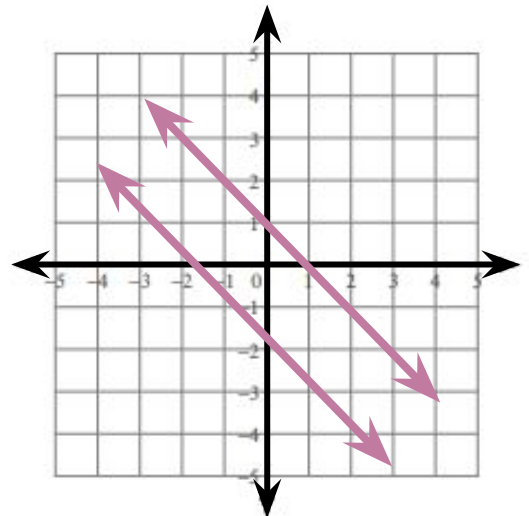
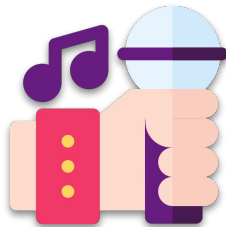


# SYSTEMS OF EQUATIONS

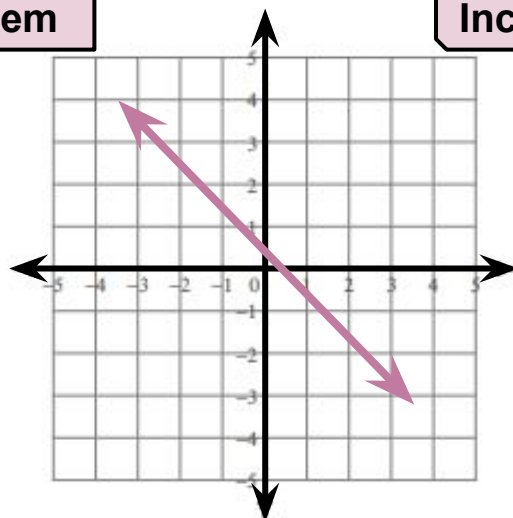
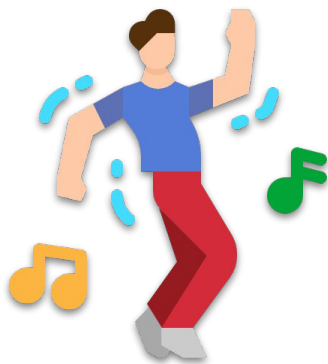
- **Independent System** - It is a system that has only one solution. The unknowns of this system has a unique value in order for the system to be true.
- **Dependent System** - It is a system that has more than one or infinitely many solutions . If you choose any values of  $x$  and  $y$ , the equations in the system will be satisfied.
- **Inconsistent System** - It is a system that has no solution. There is no pair of  $x$  and  $y$ -values that will satisfy the equations.



Independent System



Inconsistent System



Dependent System



# STEPS IN GRAPHING

Example:

$$y = -3x + 4$$

$$y = 3x - 2$$



**STEP 1** Graph the first equation.

Find the slope and y-intercept of the first equation.

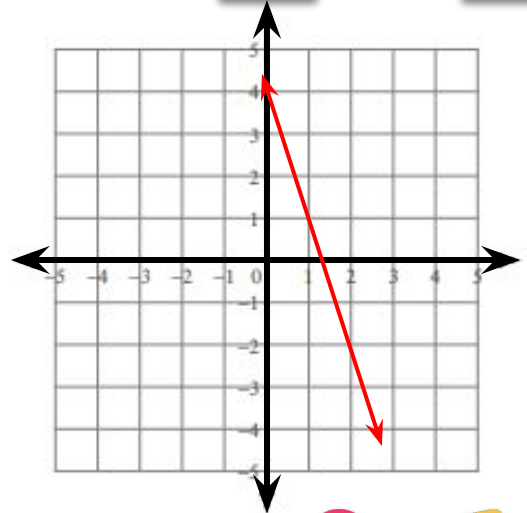
$$y = -3x + 4$$

$$Y = mx + b$$

$$m = -3$$

$$b = 4$$

Graph the line using the slope and y-intercept.



**STEP 2** Graph the second equation.

Find the slope and y-intercept of the first equation.

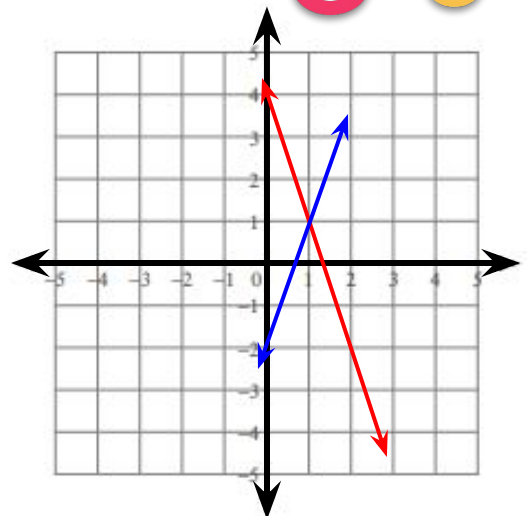
$$y = 3x - 2$$

$$Y = mx + b$$

$$m = 3$$

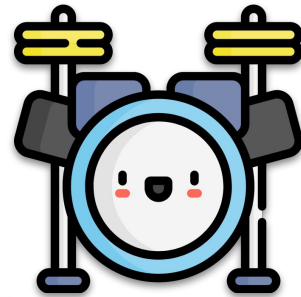
$$b = -2$$

Graph the line using the slope and y-intercept.

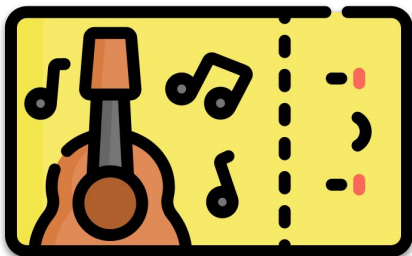
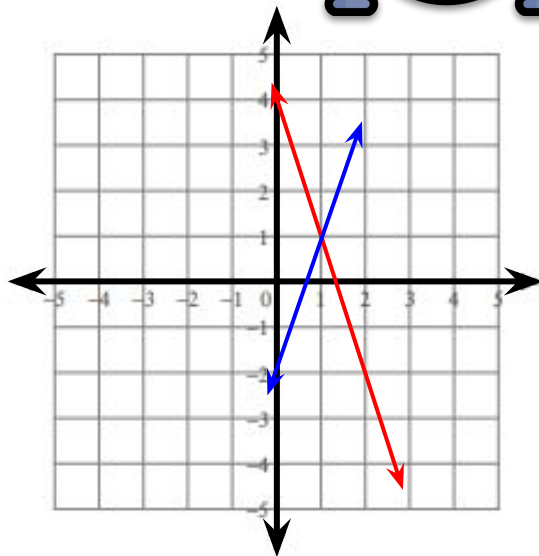


## STEPS IN GRAPHING

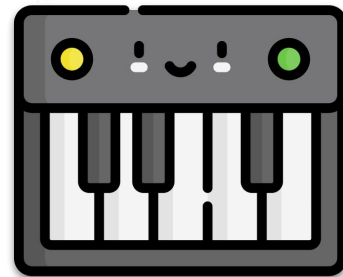
**STEP 3** Observe the graph of the lines.



Notice that the lines intersect.



**STEP 4** Identify the solution to the system.



If the lines intersect, the point of intersection is the solution set.

If the lines are parallel, the system has no solution.

If the lines are the same, the system has infinite number of solutions.

The lines intersect at point (1,1)

Equation 1  
 $y = -3x + 4$

$$1 = -3(1) + 4$$

$$1 = 1$$

Equation 2  
 $y = 3x - 2$

$$1 = 3(1) - 2$$

$$1 = 1$$



## SOLVING BY SUBSTITUTION

Example:  $y = -3x + 4$      $y = 3x - 2$



Substitute  $y$   $\Rightarrow$   $3x - 2 = -3x + 4$

Solve for  $x$   $\Rightarrow$   $3x + 3x = 4 + 2$   
 $6x = 6$   
 $x = 1$

Substitute value of  $x$  to the first equation.  $\Rightarrow$   $y = -3(1) + 4$   
 $y = 1$

Therefore, using substitution, the solution set of the given system of equations is  $(1,1)$

## SOLVING BY ELIMINATION

Example:  $y = -3x + 4$      $y = 3x - 2$

Subtract the two equations.  $\Rightarrow$  
$$\begin{array}{r} y = -3x + 4 \\ -(y = 3x - 2) \\ \hline 0 = -6x + 6 \end{array}$$

Solve for  $x$   $\Rightarrow$   $0 = -6x + 6$   
 $6x = 6$   
 $x = 1$

Substitute value of  $x$  to the first equation.  $\Rightarrow$   $y = -3(1) + 4$   
 $y = 1$

Note: If the coefficient of the variables is not the same, you can multiply a number to either of the equation in order to get the coefficients of the variables the same.

Therefore, using elimination method, the solution set of the given system of equations is  $(1,1)$



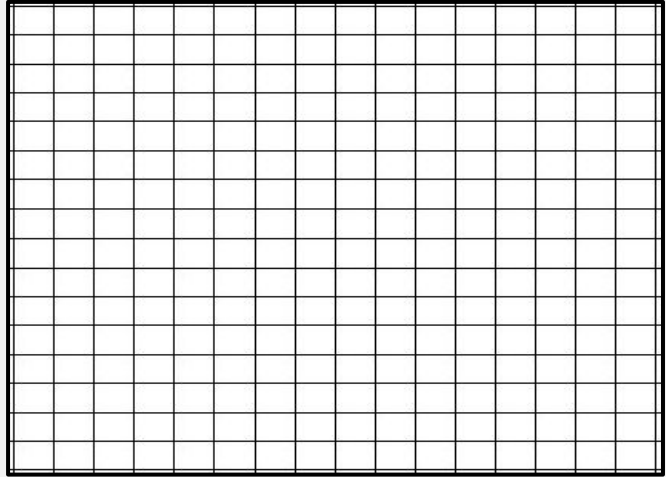
## LET'S PRACTICE

1. Determine the solution set of the given system of equations. Graph the equations to prove your answer.

$$y = x + 2$$

$$x = -3$$

Solution Set: \_\_\_\_\_



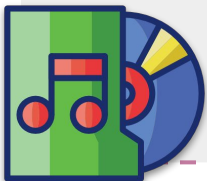
Using elimination method and substitution method, determine the solution set of the given system of equations.

$$x + 7y = 0$$

$$2x - 8y = 22$$

1. Substitution Method

2. Elimination Method



# TABLE OF ACTIVITIES

1. Rock N' Roll Stadium
2. Pop Idol Concert
3. The Country Music Club
4. Joey the Piano Man
5. The Choir Conundrum
6. Music Town Parade
7. Jazz and Jives
8. Opera Operation
9. Midnight Symphony
10. The Grand Finale

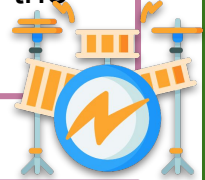


# ROCK N' ROLL STADIUM

Bang your heads to the beat! Rock it out by reading each statement carefully. Identify whether it is true or false. Write your answer on the box provided.

1

The point of intersection of two lines is the solution set of the system.



2

If the lines are parallel, the system has one solution.

3

If the lines are the same, the system has infinite number of solution.

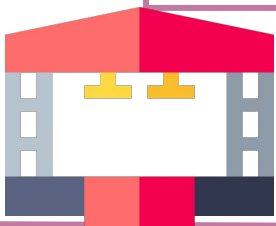
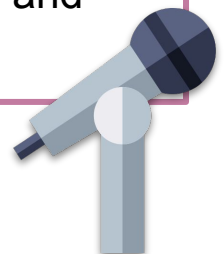


4

The solutions of a system of equations are the values of the variables that make all the equations true

5

Every point on the line is a solution to the equation and every solution to the equation is a point on the line

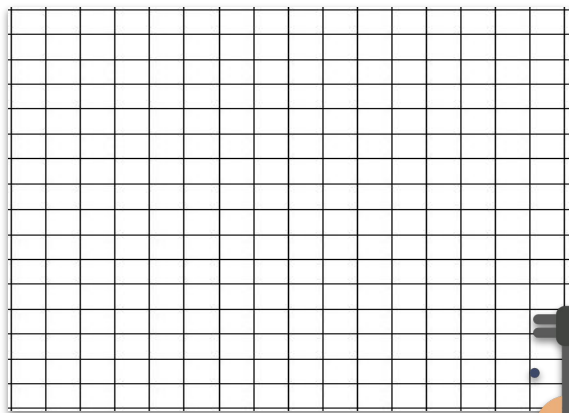




# POP IDOL CONCERT

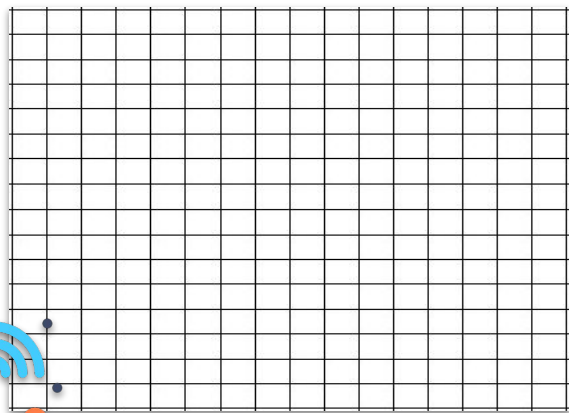
To cheer for your favorite pop idol, graph the following systems of equations below and identify whether the lines intersect, are parallel or the same. Write “intersect”, “parallel” or “same” on the line provided below.

1.)  $y = -2x + 2$        $y = -2x - 2$



\_\_\_\_\_

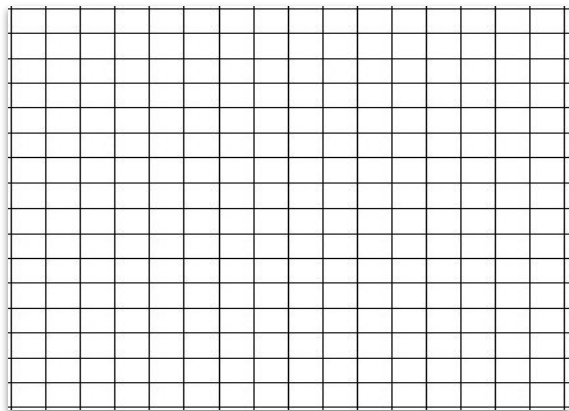
2.)  $-2x + y = -6$        $-x - y = -1$



\_\_\_\_\_

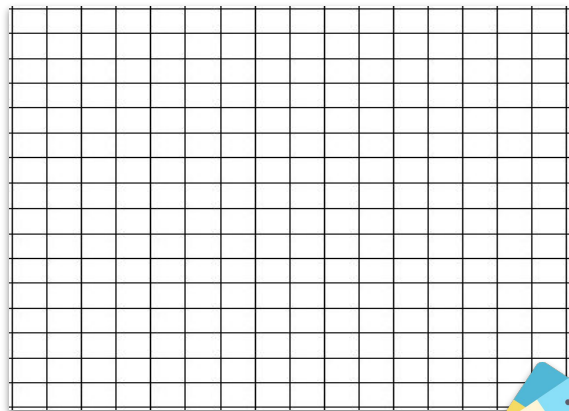


3.)  $y = 2x - 3$        $3y = 6x - 9$



\_\_\_\_\_

4.)  $y = -\frac{1}{2}x - 2$        $y = -2x + 2$



\_\_\_\_\_



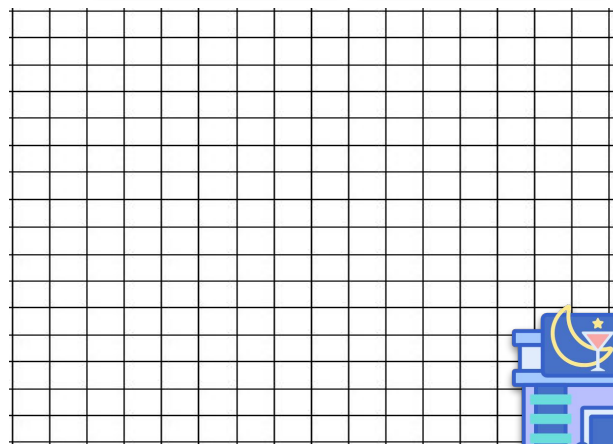
# THE COUNTRY MUSIC CLUB

To enter the country music club, answer the following. Given the system of equations below, graph the equations and identify the solution set of the system.

1.

$$y = -4x + 5$$

$$y = 5x - 4$$

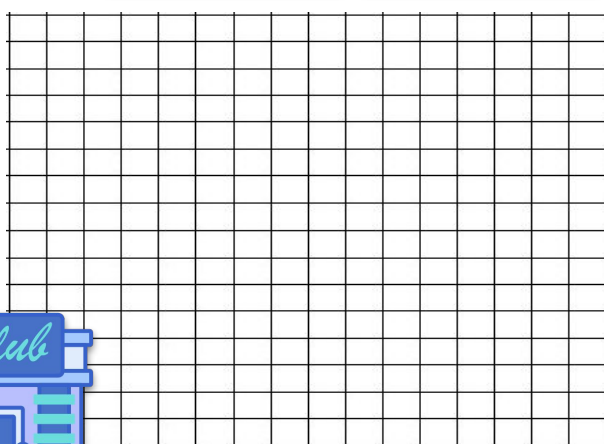


Solution set : \_\_\_\_\_

2.

$$y = 3x - 3$$

$$y = -x + 5$$



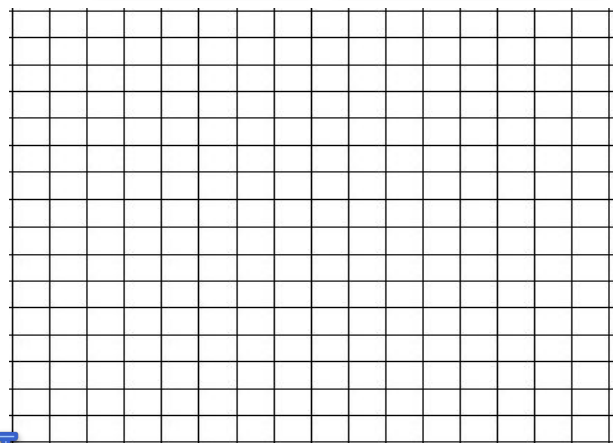
Solution set : \_\_\_\_\_



3.

$$y = 2x - 4$$

$$y = -\frac{3}{2}x + 3$$

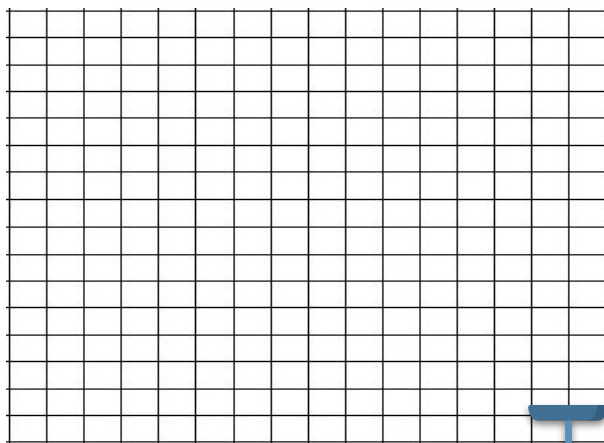


Solution set : \_\_\_\_\_

4.

$$y = -2x + 7$$

$$y = 8x - 3$$



Solution set : \_\_\_\_\_



# JOEY THE PIANO MAN

Joey can play all songs in his piano. You can request a song if you answer the following. Use substitution method to determine the solution set of the following systems of equations.

$$\begin{aligned} 1. \quad & -4x + 2y = 18 \\ & 6x + 6y = 0 \end{aligned}$$

$$\begin{aligned} 2. \quad & -2x - 2y = 10 \\ & 4x - y = 20 \end{aligned}$$



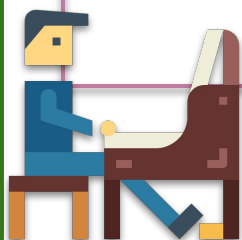
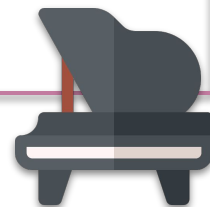
$$\begin{aligned} 3. \quad & y = x - 3 \\ & y = 4x - 9 \end{aligned}$$

$$\begin{aligned} 4. \quad & x + 3y = 9 \\ & -2x + y = -11 \end{aligned}$$



$$\begin{aligned} 5. \quad & 4x + 2y = 10 \\ & x - y = 13 \end{aligned}$$

$$\begin{aligned} 6. \quad & 7x + 2y = -19 \\ & -x + 2y = 21 \end{aligned}$$



# THE CHOIR CONUNDRUM

Oh no! A singer in the choir suddenly felt ill! As you are a fantastic singer, you can help them by determining the solution set of the following systems of equations using elimination method.



$$3x + y = 12$$

$$x = y - 8$$

1

$$6x - 12y = 24$$

$$-x - 6y = 4$$

2

$$-8x - 10y = 24$$

$$6x + 5y = 2$$

3

$$5x + 3y = 14$$

$$3x - 4y = -9$$

4

$$7x + 2y = -6$$

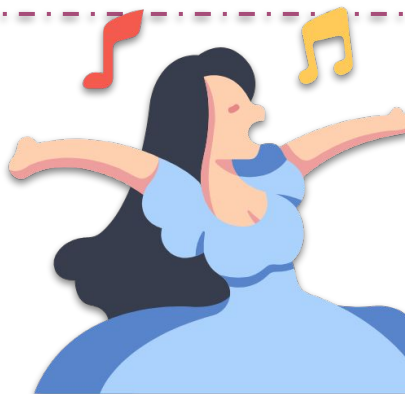
$$4x - y = 9$$

5

$$2x - 3y = 7$$

$$3x + y = 5$$

6



# MUSIC TOWN PARADE

Bugles and Drums march along the Musictown. To watch their performance, answer the following problems first. Given the system of equations, determine whether the stated solution set is correct or incorrect. Justify your answer on the space provided.

1.)

Equation 1:  $-15 + 20x - 8y = 0$

Equation 2:  $4 = -8y - 24x$

Solution Set:  $(0, -2)$

Correct or incorrect? \_\_\_\_\_

Reason:

2.)

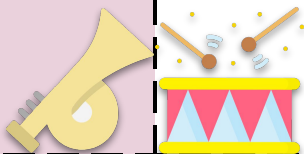
Equation 1:  $y = -4$

Equation 2:  $-3x - 6y = 15$

Solution Set:  $(3, -4)$

Correct or incorrect? \_\_\_\_\_

Reason:



3.)

Equation 1:  $y = 4x + 22$

Equation 2:  $y = -4x - 18$

Solution Set:  $(-5, -2)$

Correct or incorrect? \_\_\_\_\_

Reason:

4.)

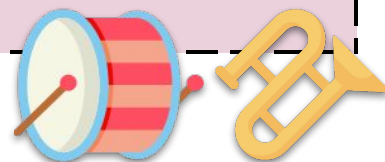
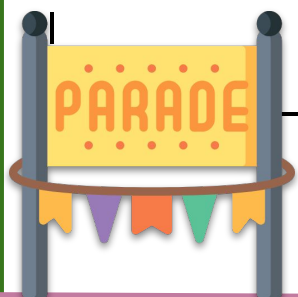
Equation 1:  $2x + 4y = -10$

Equation 2:  $7x + 8y = -23$

Solution Set:  $(-1, -2)$

Correct or incorrect? \_\_\_\_\_

Reason:



# JAZZ AND JIVES

Smooth Music comes out of the concert hall. Jive with their sax and drums by answering the following. Given the systems of equations below, determine the solution set using substitution method and elimination method. Write your solutions on the space provided.

$$\begin{aligned}x - 3y &= -3 \\x + y &= 5\end{aligned}$$



$$\begin{aligned}x - 2y &= 4 \\3y + x &= 19\end{aligned}$$



$$\begin{aligned}-8x - y &= 24 \\6x + 2y &= 2\end{aligned}$$



1. Substitution Method

2. Elimination Method

3. Substitution Method

4. Elimination Method

5. Substitution Method

6. Elimination Method



# OPERA OPERATION

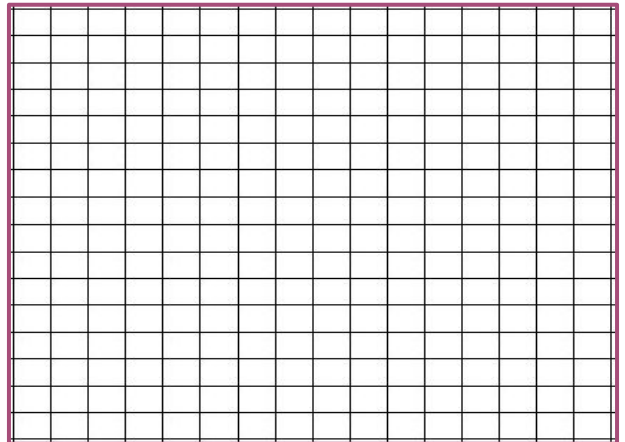
You listened to the angelic voices of the singers. To watch the aria, identify another equation that will make the given solution set correct. Graph the equations to prove your answer.

1

Equation 1:  $y = -2x + 3$

Solution Set:  $(3, -4)$

Equation 2: \_\_\_\_\_

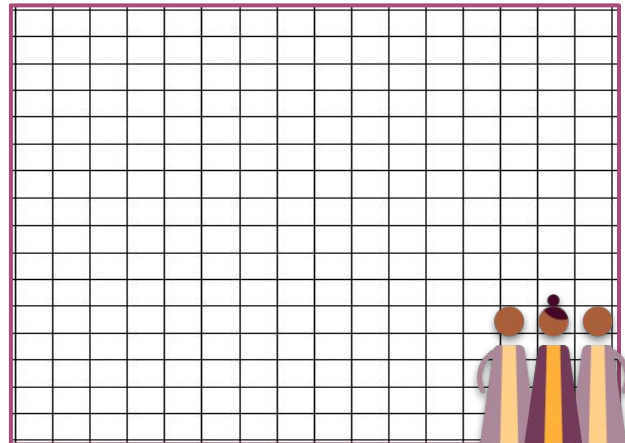
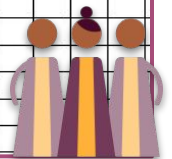


2

Equation 1:  $y = x + 2$

Solution Set:  $(4, 1)$

Equation 2: \_\_\_\_\_

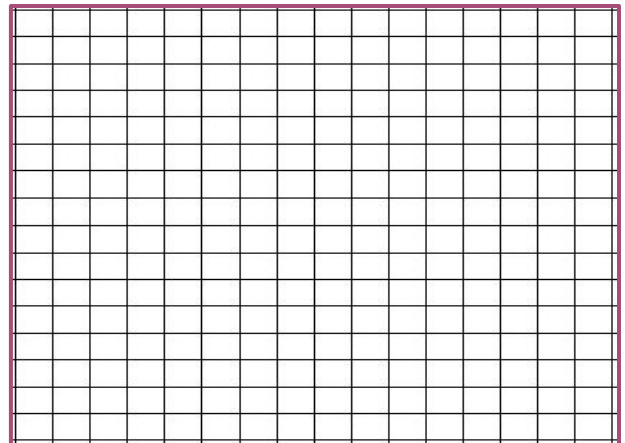
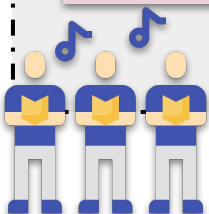


3

Equation 1:  $y = -\frac{2}{3}x + 1$

Solution Set:  $(2, -2)$

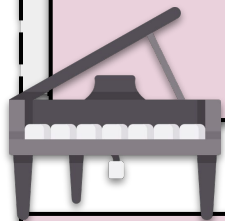
Equation 2: \_\_\_\_\_



# MIDNIGHT SYMPHONY

You hear a Symphony that is captivating and the same time chilling. To enjoy the performance, write a system of equations with the solution sets given below.

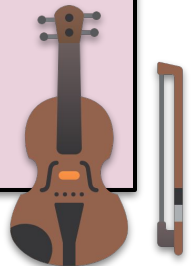
1.  $(2, -1)$



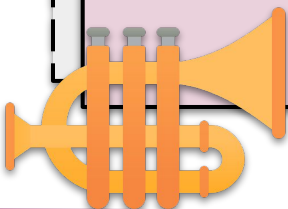
2.  $(5, 3)$

3.  $(3, 2)$

4.  $(-2, 4)$



5.  $(6, -3)$



6.  $(4, 3)$

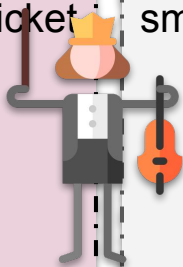




# THE GRAND FINALE

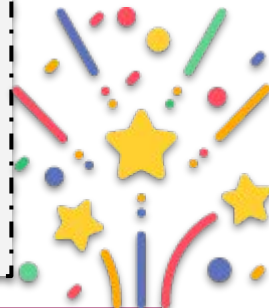
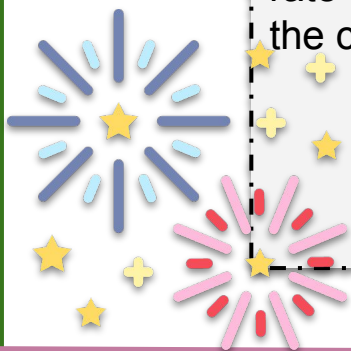
The Overture playing over the fireworks as the day ends in Musictown. To watch their performance, read and answer each word problem carefully. Show your solution on the space provided.

1. Anna bought tickets for the upcoming classical concert. She bought 2 VIP and 3 General Admission tickets for \$210 for her family. The next day, She also bought tickets for her friends, 1 VIP and 2 General Admission for \$120. How much is the VIP ticket price and the General admission ticket price?



2. Sam is selling lightsticks in the 2-day Rock n' Roll Concert . On the first day, he sold 4 big lightsticks and 6 small lightsticks and earned \$54. On the next day, he sold 8 lightsticks and 3 lightsticks, he earned \$63. How much is the cost of the big and small lightsticks?

3. John will attend a Music Festival that will be conducted in an island. It takes 2 hours for a boat to travel 16 miles downstreams going to the island, and 8 hours to return. What is the rate of the boat in still water and the rate of the current?

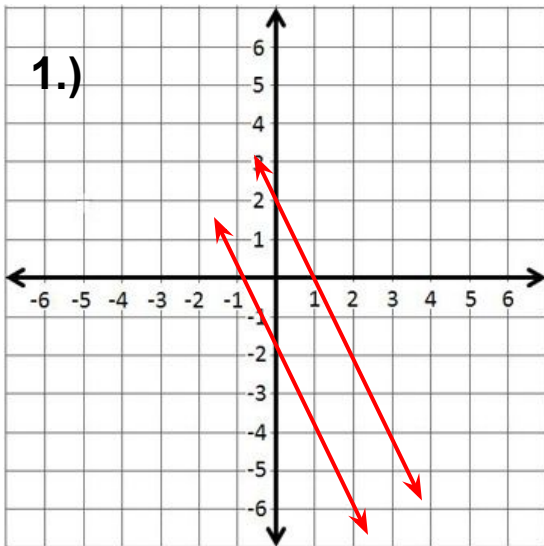


# ANSWER GUIDE

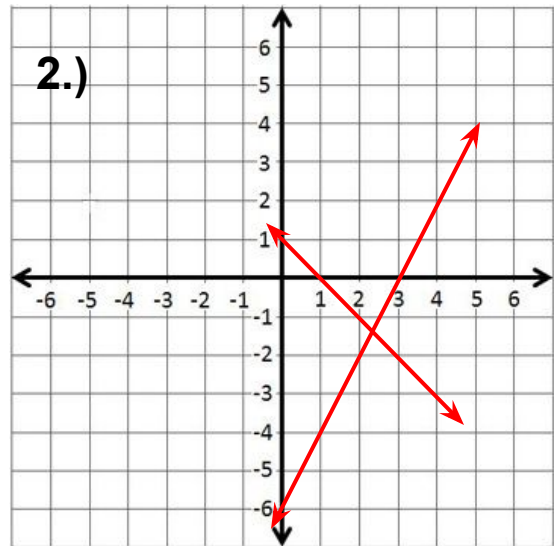
## Activity 1

1. True    2. False    3. True    4. True    5. True

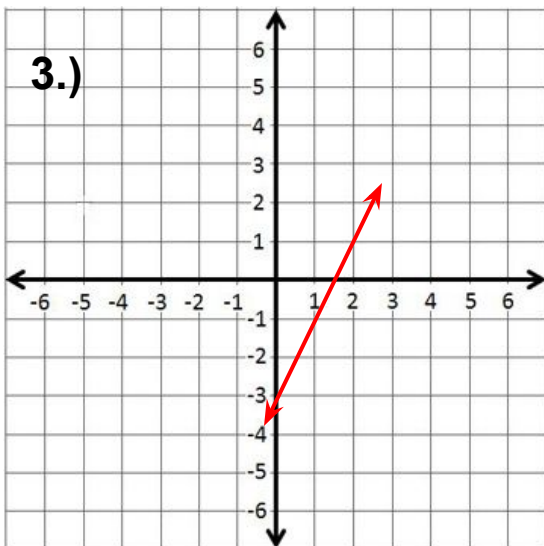
## Activity 2



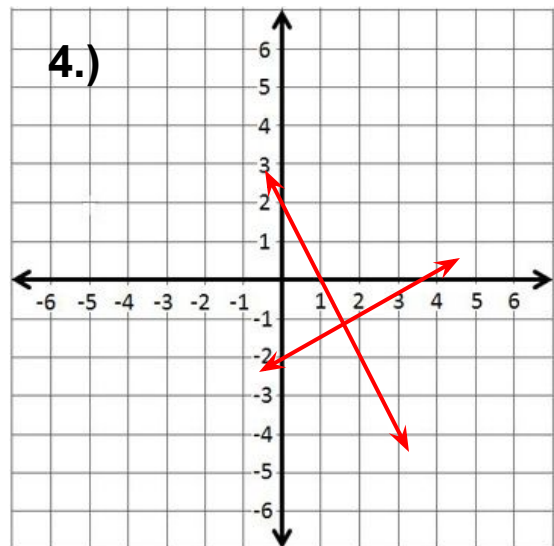
parallel



intersect



same

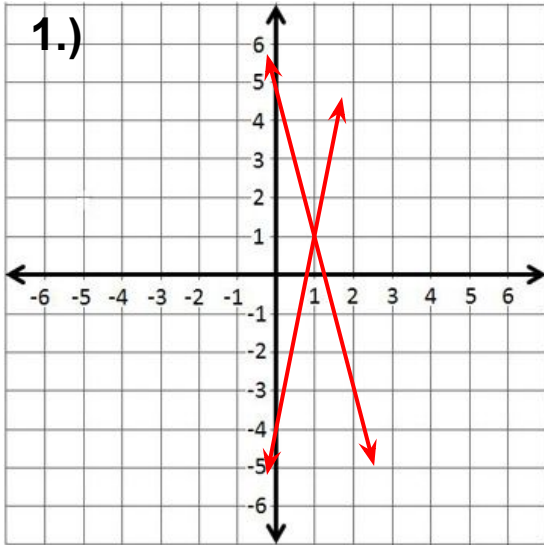


intersect

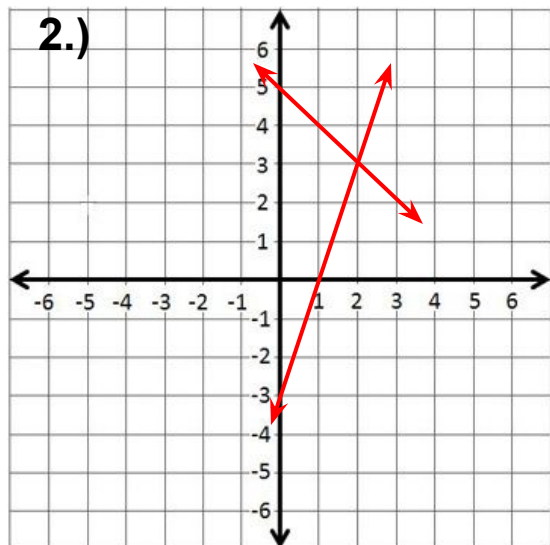


# ANSWER GUIDE

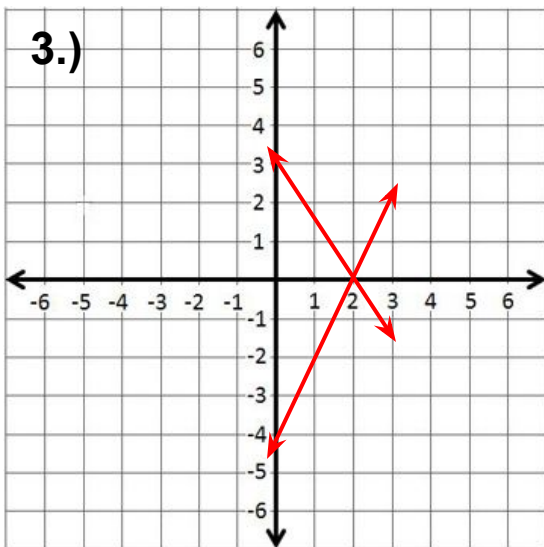
## Activity 3



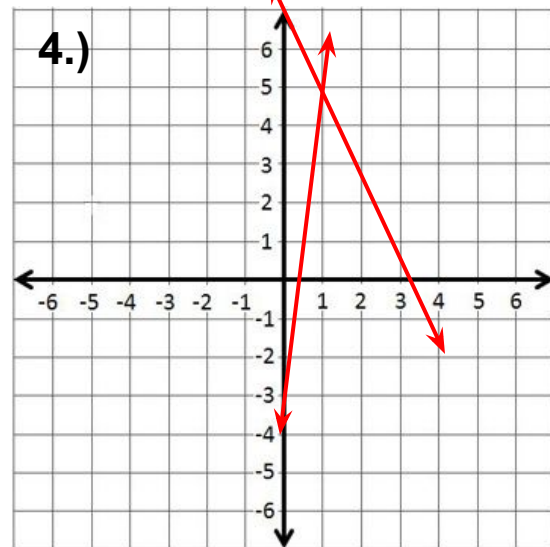
(1, 1)



(2, 3)



(2, 0)



(1, 5)



# ANSWER GUIDE

## Activity 4

1.)  $(\frac{1}{2})(-4x + 2y = 18)(\frac{1}{2})$

$$-2x + y = 9$$

$$y = 2x + 9$$

Substitute:

$$6x - 6(2x + 9) = 0$$

$$6x - 12x - 54 = 0$$

$$-6x = 54$$

$$x = -9$$

$$y = 2(-9) + 9$$

$$y = -9$$

**Solution set: (-9, -9)**

3.)  $y = x - 3$  ;  $y = 4x - 9$

$$x - 3 = 4x - 9$$

$$x - 4x = -9 + 3$$

$$-3x = -6$$

$$x = 2$$

$$y = 4(2) - 9$$

$$y = -1$$

**Solution set: (2, -1)**

5.)  $4x + 2y = 10$

$$y = -2x + 5$$

$$-2x + 5 = x - 13$$

$$-2x - x = -13 - 5$$

$$-3x = -18$$

$$x = 6$$

$$6 - y = 13$$

$$y = -7$$

**Solution set: (6, -7)**

2.)  $(\frac{1}{2})(-2x - 2y = 10)(\frac{1}{2})$

$$-x - y = 5$$

$$y = -x - 5$$

Substitute:

$$4x - (-x - 5) = 20$$

$$4x + x + 5 = 20$$

$$5x = 15$$

$$x = 3$$

$$y = -(3) - 5$$

$$y = -8$$

**Solution set: (3, -8)**

4.)  $x + 3y = 9$

$$y = -\frac{1}{3}x + 3$$

$$-2x + (-\frac{1}{3}x + 3) = -11$$

$$-2x - \frac{1}{3}x + 3 = -11$$

$$-\frac{7}{3}x = -14$$

$$x = 6$$

$$6 + 3y = 9$$

$$y = 1$$

**Solution set: (6, 1)**

6.)  $7x + 2y = -19$

$$y = -\frac{7}{2}x - \frac{19}{2}$$

$$-\frac{7}{2}x - \frac{19}{2} = \frac{1}{2}x + \frac{21}{2}$$

$$-\frac{7}{2}x - \frac{1}{2}x = \frac{21}{2} + \frac{19}{2}$$

$$-4x = 20$$

$$x = -5$$

$$-(5) + 2y = 21$$

$$y = 8$$

**Solution set: (-5, 8)**



# ANSWER GUIDE

## Activity 5

1.)  $(-1)(3x + y = 12)(-1)$

$$-3x - y = -12$$

$$-3x - y = -12$$

$$- \quad (x - y = -8)$$

$$\hline -4x - 0 = -4$$

$$-4x = -4 ; x = 1$$

$$1 = y - 8 ; y = 9$$

**Solution set: (1, 9)**

3.)  $(-2)(6x + 5y = 2)(-2)$

$$-12x - 10y = -4$$

$$-12x - 10y = -4$$

$$-(-8x - 10y = 24)$$

$$\hline -4x - 0 = -28$$

$$-4x - 0 = -28$$

$$-4x = -28 ; x = 7$$

$$-8(7) - 10y = 24 ; y = -8$$

**Solution set: (7, -8)**

5.)  $(-2)(4x - y = 9)(-2)$

$$-8x + 2y = -18$$

$$-8x + 2y = -18$$

$$-(7x + 2y = -6)$$

$$\hline -15x - 0 = -12$$

$$-15x = -12 ; x = \frac{4}{5}$$

$$4(\frac{4}{5}) - y = 9 ; y = -\frac{29}{5}$$

**Solution set: ( $\frac{4}{5}$ ,  $-\frac{29}{5}$ )**

2.)  $(2)(-x - 6y = 4)(2)$

$$-2x - 12y = 8$$

$$-2x - 12y = 8$$

$$- \quad (6x - 12y = 24)$$

$$\hline -8x - 0 = -16$$

$$-8x = -16 ; x = 2$$

$$-2 - 6y = 4 ; y = -1$$

**Solution set: (2, -1)**

4.)  $(-4)(5x + 3y = 14)(-4)$

$$-20x - 12y = -56$$

$$(3)(3x - 4y = -9)(3)$$

$$9x - 12y = -27$$

$$-20x - 12y = -56$$

$$- \quad (9x - 12y = -27)$$

$$\hline -29x - 0 = -29$$

$$-29x = -29 ; x = 1$$

$$3(1) - 4y = -9 ; y = 3$$

**Solution set: (1, 3)**

6.)  $(-3)(3x + y = 5)(-3)$

$$-9x - 3y = -15$$

$$-9x - 3y = -15$$

$$-(2x - 3y = 7)$$

$$\hline -11x - 0 = -22$$

$$-11x = -22 ; x = 2$$

$$3(2) + y = 5 ; y = -1$$

**Solution set: (2, -1)**



# ANSWER GUIDE

## Activity 6

1.)  $(-1)(8y + 24x = -4)(-1)$

$$-8y - 24x = 4$$

$$-8y - 24x = 4$$

$$- \frac{(-8y + 20x = 15)}{0 - 44x = -11}$$

$$0 - 44x = -11$$

$$-44x = -11 ; x = \frac{1}{4}$$

$$-15 + 20(\frac{1}{4}) - 8y = 0 ; y = -\frac{5}{4}$$

**Solution set:  $(\frac{1}{4}, -\frac{5}{4})$**

**Therefore, it is incorrect.**

3.)  $y = 4x + 22 ; y = -4x - 18$

$$4x + 22 = -4x - 18$$

$$4x + 4x = -18 - 22$$

$$8x = -40 ; x = -5$$

$$y = -4(-5) - 18 ; y = -2$$

**Solution set:  $(-5, -2)$**

**Therefore, it is correct.**

2.)  $-3x - 6y = 15$

$$-3x - 6(-4) = 15$$

$$-3x = 15 - 24$$

$$-3x = -9 ; x = 3$$

$$-3(3) - 6y = 15 ; y = -4$$

**Solution set:  $(3, -4)$**

**Therefore it is correct.**

4.)  $(2)(2x + 4y = -10)(2)$

$$4x + 8y = -20$$

$$4x + 8y = -20$$

$$- (7x + 8y = -23)$$

$$\frac{-3x - 0 = 3}{-3x - 0 = 3}$$

$$-3x = 3 ; x = -1$$

$$7(-1) + 8y = -23 ; y = -2$$

**Solution set:  $(-1, -2)$**

**Therefore, it is correct.**

## Activity 5

1.)  $y = \frac{1}{3}x + 1 ; y = 5 - x$

$$\frac{1}{3}x + 1 = 5 - x$$

$$\frac{1}{3}x + x = 5 - 1$$

$$x = 3$$

$$y = 5 - x ; y = 5 - 3$$

$$y = 2$$

**Solution set:  $(3, 2)$**

2.  $y = \frac{1}{3}x + 1$

$$- (y = -x + 5)$$

$$\frac{0 = \frac{4}{3}x - 4}{4/3 x = 4}$$

$$4/3 x = 4$$

$$x = 3$$

$$y = 5 - x ; y = 5 - 3$$

$$y = 2$$

**Solution set:  $(3, 2)$**



## ANSWER GUIDE

3.)  $y = \frac{1}{2}x - 2$  ;  $y = -\frac{1}{3}x + 19/3$

$$\frac{1}{2}x - 2 = -\frac{1}{3}x + 19/3$$

$$\frac{1}{2}x + \frac{1}{3}x = 19/3 + 2$$

$$\mathbf{x = 10}$$

$$y = \frac{1}{2}x - 2 ; y = \frac{1}{2}(10) - 2$$

$$\mathbf{y = 3}$$

**Solution set: (10, 3)**

5.)  $y = -8x - 24$  ;  $y = -3x + 1$

$$-8x - 24 = -3x + 1$$

$$-8x + 3x = 1 + 24$$

$$\mathbf{x = -5}$$

$$y = -8x - 24 ; y = -8(-5) - 24$$

$$\mathbf{y = 16}$$

**Solution set: (-5, 16)**

4.  $y = \frac{1}{2}x - 2$

$$- (y = -\frac{1}{3}x + 19/3)$$

$$0 = 5/6x - 4$$

$$5/6x = 25/3$$

$$\mathbf{x = 10}$$

$$y = \frac{1}{2}x - 2 ; y = \frac{1}{2}(10) - 2$$

$$\mathbf{y = 3}$$

**Solution set: (10, 3)**

6.)  $y = -8x - 24$

$$- (y = -3x + 1)$$

$$0 = -5x - 25$$

$$5x = -25$$

$$\mathbf{x = -5}$$

$$y = -8x - 24 ; y = -8(-5) - 24$$

$$\mathbf{y = 16}$$

**Solution set: (-5, 16)**

### Activity 8

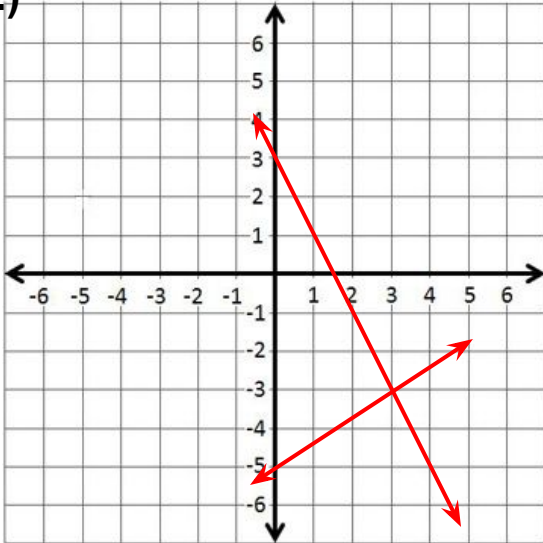
Learner's answers may vary.



# ANSWER GUIDE

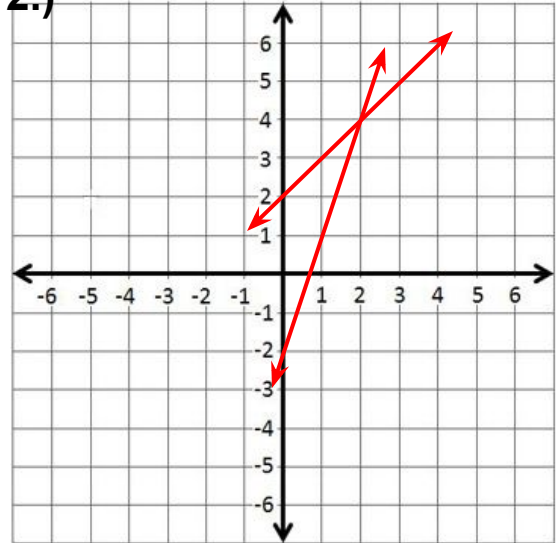
## Activity 9

1.)



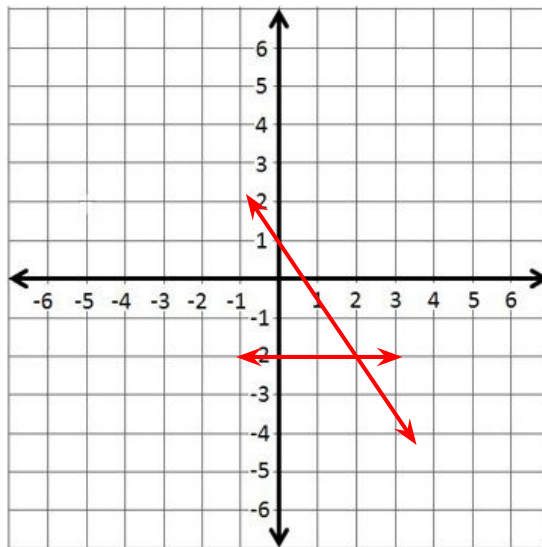
Equation 1:  $y = -\frac{2}{3}x + 4$   
Equation 2:  $y = \frac{2}{3}x - 5$

2.)



Equation 1:  $y = -x + 2$   
Equation 2:  $y = 3x - 2$

3.)



Equation 1:  $y = -2$   
Equation 2:  $y = -2x + 2$





# ANSWER GUIDE

## Activity 10

1.) Let  $a$  = cost of Gen. Admission

$b$  = cost of VIP ticket

Equation 1:  $3a + 2b = \$210$

Equation 2:  $2a + b = \$120$

Solve for  $a$  and  $b$ :

Using substitution method

Equation 1:  $a = -\frac{2}{3}b + 70$

$2(-\frac{2}{3}b + 70) + b = 120$

$-\frac{4}{3}b + 140 + b = 120$

$-\frac{1}{3}b = 120 - 140$  ;  **$b = 60$**

Substitute  $b$  to equation 2

$2a + (60) = 120$  ;  **$a = 30$**

Therefore

**Gen. Admission ticket = \$30**

**VIP ticket = \$60**

3.) Let  $B$  = speed of the boat

$C$  = speed of the current

Equation 1:  $16 = 2(B + C)$

Equation 2:  $16 = 8(B - C)$

Solve for  $B$  and  $C$ :

Using substitution method

Equation 1:  $B = -C + 8$

$16 = 8(-C + 8) - 8C$

$16 = -8C + 64 - 8C$

$16C = 64 - 16$  ;  **$C = 3$**

Substitute  $C$  to equation 2

$16 = 8(B - 3)$  ;  **$B = 5$**

Therefore

**speed of the boat = 5mph**

**speed of the current = 3mph**

2.) Let  $x$  = cost of big lightsticks

$y$  = cost of small lightsticks

Equation 1:  $4x + 6y = \$54$

Equation 2:  $8x + 3y = \$63$

Solve for  $x$  and  $y$ :

Using substitution method

Equation 1:  $y = -\frac{4}{6}x + 9$

$8x + 3(-\frac{4}{6}x + 9) = 63$

$8x - 2x + 27 = 63$

$6x = 63 - 27$  ;  **$x = 6$**

Substitute  $x$  to equation 2

$8(6) + 3y = 63$  ;  **$y = 5$**

Therefore

**Big lightsticks = \$6**

**Small lightsticks = \$5**



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