



Helping With Math

Understanding and Solving One-Variable Inequalities

GRADE 6



Inequalities are mathematical expressions that use the symbols $>$, $<$, \geq and \leq . Solving inequalities means to find a range or set of values that an unknown variable can take and can still satisfy the given inequality.



Oliver bought a gallon of his favorite apple juice that costs \$ 4.5. If he has \$35, what is the maximum number of gallons of apple juice can he buy?



INEQUALITIES SYMBOLS

SYMBOL	WORDS	EXAMPLE
$<$	<i>less than</i>	$a < 5$
$>$	<i>greater than</i>	$x > -10$
\leq	<i>less than or equal to</i>	$2r \leq 1.5$
\geq	<i>greater than or equal to</i>	$y \geq 2y + 1$

RULES IN INEQUALITIES

The rules of equations and inequalities are almost alike. There are just few rules that we need to be reminded of when dealing with inequalities. Please take note of the following key points.



KEY POINTS



1. If you **add and subtract** the same number to both sides of an inequality, the inequality remains true.
2. If you multiply or divide both sides of an inequality by the same **positive number**, the inequality remains true.
3. But if you multiply or divide both sides of an inequality by a **negative number**, the inequality is no longer true. In fact, the **inequality becomes reversed**.



ILLUSTRATIVE EXAMPLES

Given: $2x - 5 > x + 3$

Solution:

$$2x - 5 > x + 3$$

$$2x - x > 3 + 5$$

$$x > 8$$



Given: $2x - 6 > 5x + 3$

Solution:

$$2x - 6 > 5x + 3$$

$$2x - 5x > 3 + 6$$

$$-3x > 9 \text{ (divide both sides by } -3\text{)}$$

$$x < -3$$

Note that in second given, where $-3x > 9$ is divided by -3 , notice that the inequality symbol was reversed. It changed from greater than to less than.

GRAPH OF INEQUALITIES

For what values of x will $x + 3 > 2$ be TRUE?



Solution:

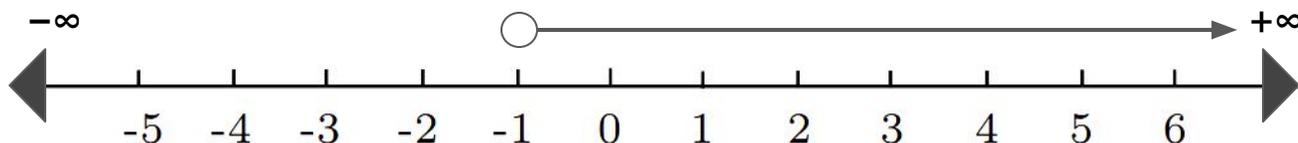
$$x + 3 > 2$$

$$x > 2 - 3$$

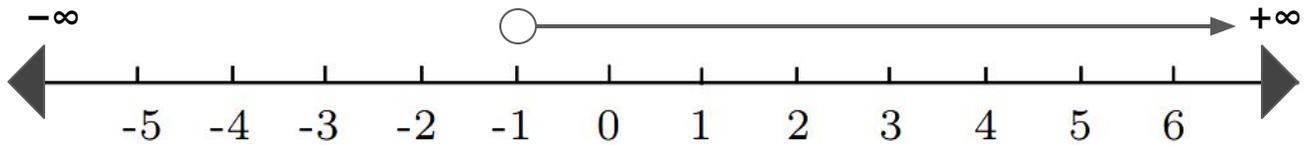
$$x > -1$$

Having $x > -1$ as an answer means that the values of x that will make the inequality $x + 3 > 2$ true numbers that are greater than -1 such as 0 , 1 , 2 , 3 , and so on.

Given this, how do we show these values using a graph? We are going to use a number line!



ILLUSTRATIVE EXAMPLES



This graph shows that the solution to the given inequality is any real numbers that is greater than -1. Thus, 0, 1, 2, 3, 4, etc. is a solution. On the contrary, -2 cannot be a solution because it is less than -1.

***Note:** Use ○ if the inequality symbol used is $<$ or $>$.
Use ● if the inequality symbol used is \leq or \geq .



For what values of k will $2k + 6 \leq 2$ be TRUE?

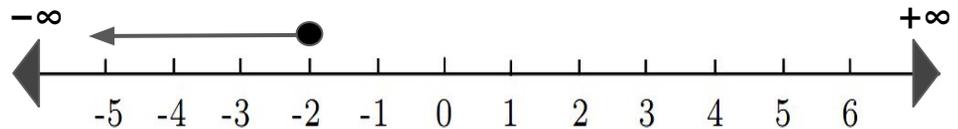
Solution:

$$2k + 6 \leq 2$$

$$2k \leq 2 - 6$$

$$2k \leq -4$$

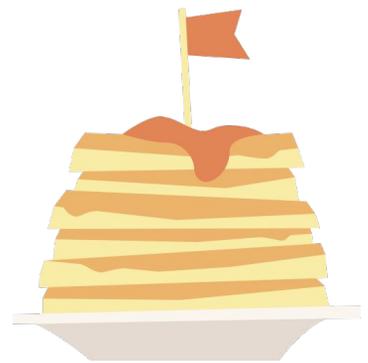
$$k \leq -2$$



WRITING SOLUTIONS

We can write the solution of an inequality using:

- Simple inequality
- Graph in a number line
- Set builder notation
- Interval notation



WRITING SOLUTIONS

Set-Builder Notation



It describes or defines the elements of a set instead of listing the elements.

Instead of writing numbers that are greater than 0, such as 1, 2, 3, 4, 5, 6, ..., we can use the set-builder notation to describe the set of solution.



$\{x \mid x > 0\}$ which is read as
"the set of x -values such that x is greater than 0."

Interval Notation



It is a way to describe continuous sets of real numbers by the numbers that bound them.

Instead of writing numbers that are greater than 0, such as 1, 2, 3, 4, 5, 6, ..., we can use interval notation to describe the solution.

$$(0, +\infty)$$

In "Interval Notation" we just write the beginning and ending numbers of the interval, and use:

- [] a square bracket when we want to **include** the end value, or
- () a round bracket when we **don't**

Interval	Inequality	Meaning
$(0, +\infty)$	$x > 0$	x is greater than 0
$[0, +\infty)$	$x \geq 0$	x is greater than or equal to 0



WRITING SOLUTIONS

Interval	Inequality	Meaning
$(-\infty, 0)$	$x < 0$	x is less than 0
$(-\infty, 0]$	$x \leq 0$	x is less than or equal to 0

PRACTICE EXERCISES

Solve the following inequalities. Represent the solution using graph, set-builder notation, and interval notation.

1. $x + 9 < 21$

2. $-2r + 4 \geq 12$



TABLE OF ACTIVITIES

1. Pancake or Cereals?
2. My Beverage Choice
3. Less Sugar vs More Sugar
4. Match Made in Kitchen
5. The Breakfast Menu
6. Coffee Break
7. Cracking the Egg
8. Food-to-Go
9. Set, Build, Eat
10. Food Escapade



PANCAKES OR CEREALS

Looks like Harley is having a hard time choosing which breakfast to eat. Is it pancakes or cereals? Write PANCAKES if the statement is TRUE, otherwise CEREALS. The breakfast choice with more number of frequency wins!

1. _____	Inequalities are mathematical expressions that use the symbols $>$, $<$, \geq and \leq .
2. _____	If you add and subtract the same number to both sides of an inequality, the inequality changes.
3. _____	If you multiply or divide both sides of an inequality by a negative number, the inequality is reversed.
4. _____	Set-builder notation describes or defines the elements of a set instead of listing the elements.
5. _____	$(-5, +\infty)$ means x is less than -5 .

My breakfast is

_____.



MY BEVERAGE CHOICE

While drinking a beverage of your choice, try to solve the following inequalities. Show your solution.

1. $5s + 7 > 2$

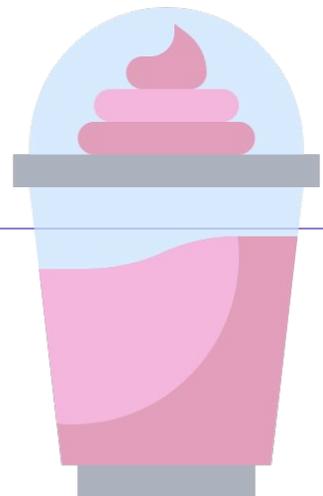
2. $4a + 6 < 3a + 7$

3. $3m - 5 \leq 3 - m$

4. $1 + 5x \geq 19$

5. $2 - 3p < -4$

6. $-3x \geq 12$



LESS SUGAR VS MORE SUGAR

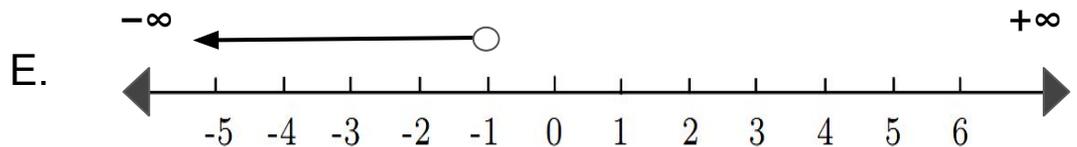
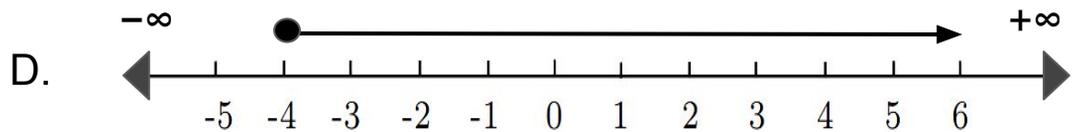
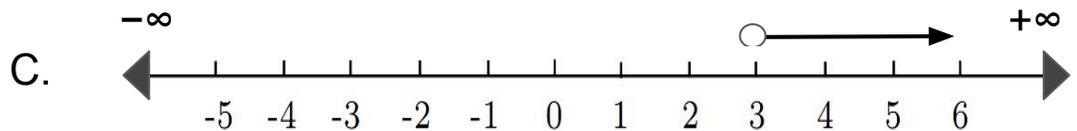
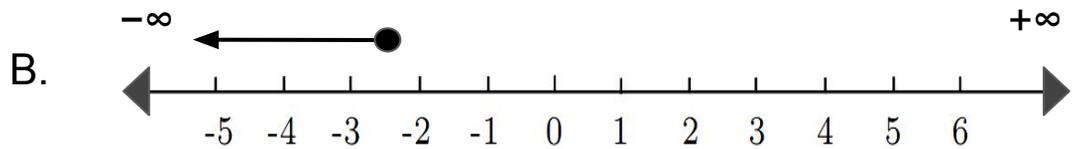
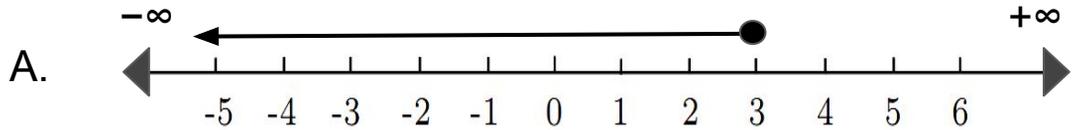
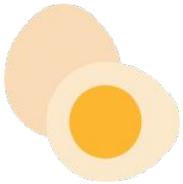
Nancy cannot decide if she will have less or more sugar for her tea. Can you help her to decide by graphing the solution of the following inequalities?

1. $5s + 7 > 2$	
2. $4a + 6 < 3a + 7$	
3. $3m - 5 \leq 3 - m$	
4. $1 + 5x \geq 19$	
5. $2 - 3p < -4$	
6. $-3x \geq 12$	



MATCH MADE IN KITCHEN

These are some of the ingredients of our favorite breakfast meals made in the kitchen. Can you help Ben to match the inequalities to its corresponding graph? Write your answer in the box.



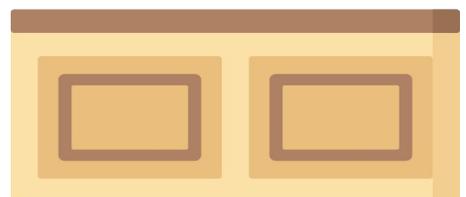
1. $x > 3$

2. $x < -1$

3. $x \geq -4$

4. $x \leq -2.5$

5. $x \leq 3$



THE BREAKFAST MENU

The breakfast menu for today offers a lot of choices of food to eat! From the given choices below, encircle which number can satisfy the following inequalities.

1. $x < 5$

-2 5 6 10

2. $3x > x + 10$

10 5 -5 0

3. $100 \leq 5x + 5$

18 19 -1 12

4. $12 < -4x$

-4 -3 -2 -7

5. $-3 < x < 7$

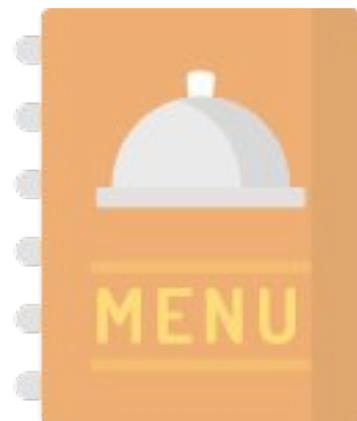
-5 -10 0 8

6. $20 - 12 \geq x$

6 9 12 10

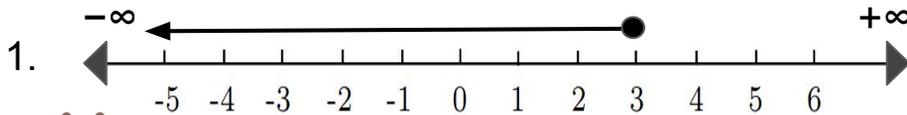
7. $3x + 1 > x + 5$

-2 0 1 3

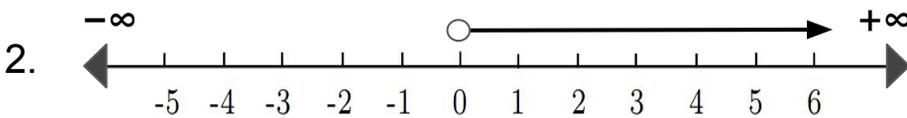


COFFEE BREAK

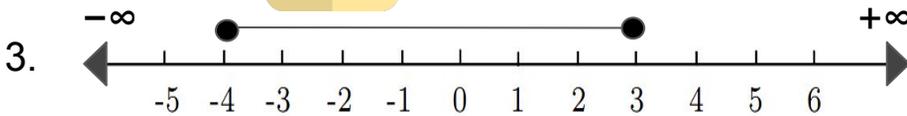
It's time for coffee break! Write the interval notation that is being presented on each graph.



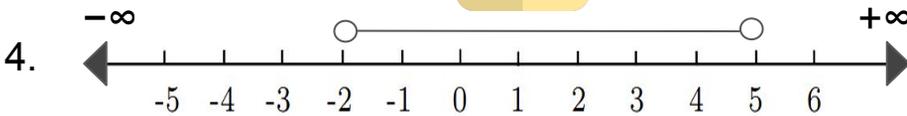
Interval notation:



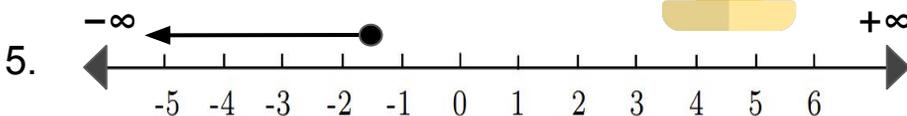
Interval notation:



Interval notation:



Interval notation:



Interval notation:



CRACKING THE EGG

How many eggs can you crack to make your favorite scrambled egg? Solve the following inequalities to know how many!

1. $48 + 6x < 100 - x$

2. $3(x - 2) > 10(x + 7)$

3. $1.5x + 3.5x < 10$

4. $\frac{1}{2} + \frac{3}{5} \geq x + 1$

5. $20 + 3x > 2(x - 5) + x$



FOOD TO GO

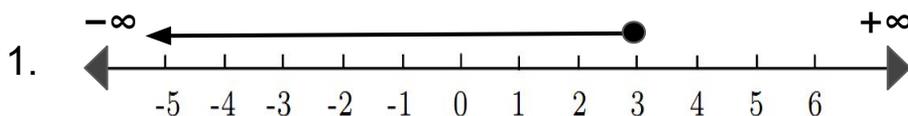
Need some snacks but cannot go out? Have some food-to-go! But before that, fill in the missing information.

Interval	Inequality	Meaning
$(-2, +\infty)$	1.	2.
3.	$x \geq 10$	4.
5.	6.	x is less than 10 but greater than 5.
7.	$20 < X < 15$	8.
9.	10.	x is greater than or equal to 21.

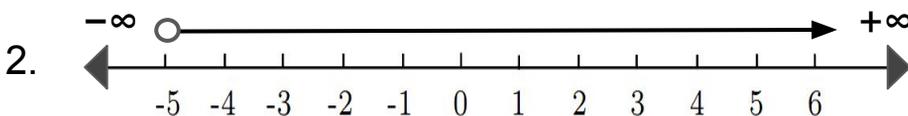


SET, BUILD, EAT

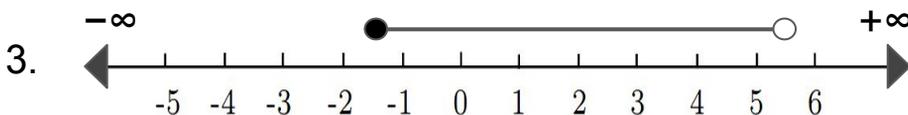
Clark and his friends are hungry but they cannot eat unless they finished with their homework. Help them solve the problems! Write a set-builder notation to represent the solution of the given inequalities.



Set-builder notation



Set-builder notation



Set-builder notation

4. x is greater than or equal to -7 .

5. x is less than -8 .

6. x is greater than 3 but less than 5 .

7. x is less than or equal to 40 .



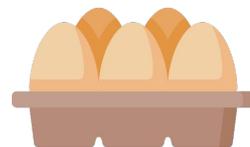
FOOD ESCAPE

These are common scenarios in a grocery store when buying food and beverages. Solve the following problems.

1. Oliver bought a gallon of his favorite apple juice that costs \$ 4.5. If he has \$35, what is the maximum number of gallons of apple juice can he buy?



2. A dozen of egg costs \$1. 54. If x represents the number of dozen of eggs that can be purchased given a particular amount, write an inequality that will show the number f dozen of eggs that can be purchased at \$ 20 to \$ 35.



3. Draw the graph that shows the solution of item no. 1.

4. Represent the solution of item no. 2 using interval notation.



ANSWER GUIDE

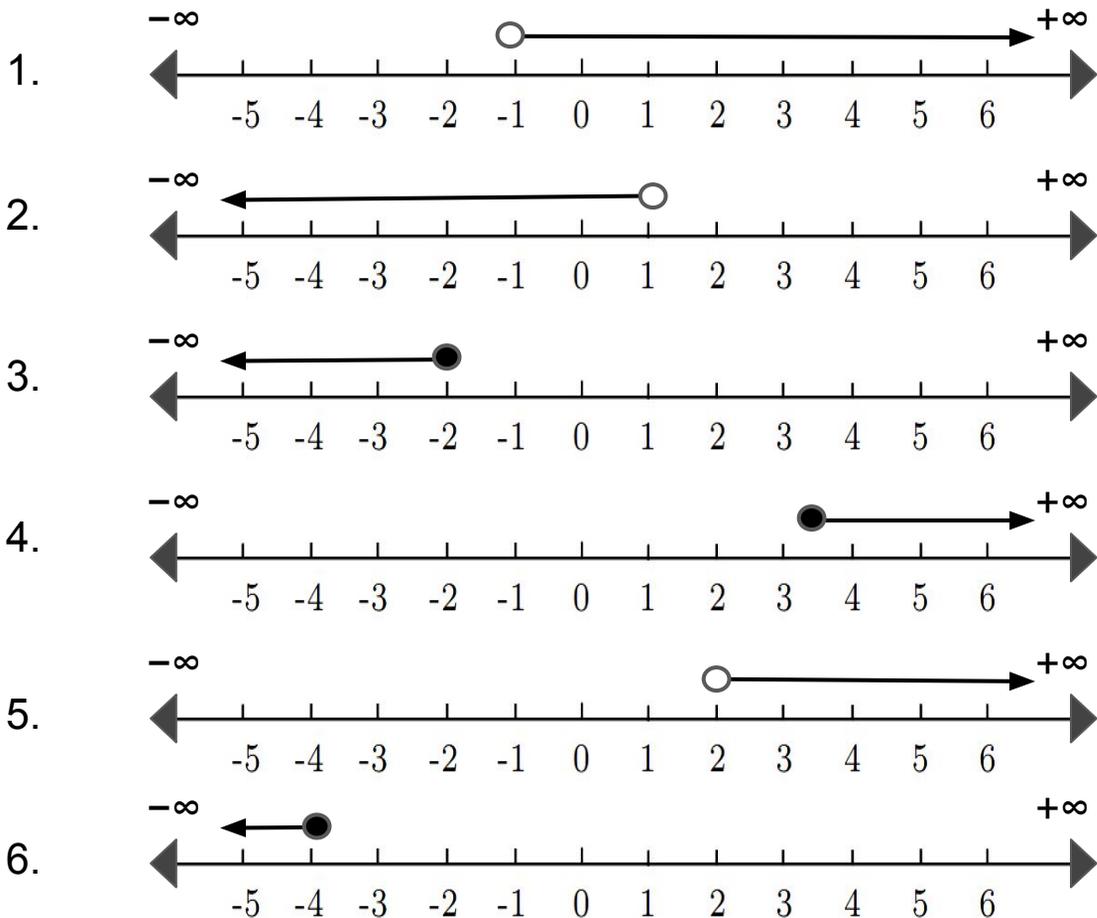
Activity 1

- | | | |
|-------------|------------|---------------------------------|
| 1. Pancakes | 2. Cereals | 3. Pancakes |
| 4. Pancakes | 5. Cereals | The breakfast will be pancakes. |

Activity 2

- | | | |
|------------------|------------|----------------|
| 1. $s > -1$ | 2. $a < 1$ | 3. $m \leq 2$ |
| 4. $x \geq 18/5$ | 5. $p > 2$ | 6. $x \leq -4$ |

Activity 3



ANSWER GUIDE

Activity 4

1. C 2. E 3. E 4. B 5. A

Activity 5

1. -2 2. 10 3. 19 4. -2
5. 0 6. 6 7. 3

Activity 6

1. $(-\infty, 3]$ 2. $(0, +\infty)$ 3. $[-4, 3]$
4. $(-2, 5)$ 5. $(-\infty, -1.5]$

Activity 7

1. $x < 7.43$ 2. $x > -10.86$ 3. $x < 2$
4. $x \leq 1/10$ 5. No solution.

Activity 8

1. $x > -2$ 2. X is greater than -2. 3. $[10, +\infty)$
4. X is greater than or equal to 10 5. $(5, 10)$ 6. $5 < x < 10$
7. $(15, 20)$ 8. X is greater than 15 but less than 20
9. $[21, +\infty)$ 10. $x \geq 21$



ANSWER GUIDE

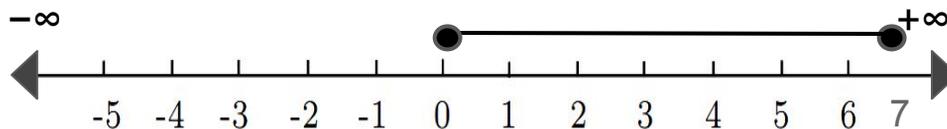
Activity 9

1. $\{x \mid x \leq 3\}$
2. $\{x \mid x > -5\}$
3. $\{x \mid -1.5 \leq x < 5.5\}$
4. $\{x \mid x \geq -7\}$
5. $\{x \mid x < -8\}$
6. $\{x \mid 3 < x < 5\}$
7. $\{x \mid x \leq 40\}$

Activity 10

1. 7 gallons
2. $12 \leq x \leq 22$

3.



4. $[12, 22]$



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