



# Helping With Math

## Understanding and Solving One-Variable Equations

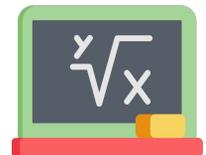
GRADE 6



In algebra, an equation can be defined as a mathematical sentence composed of an equal symbol between two algebraic expressions that have the same value.



### Equation vs Expression



Equation	Expression
It is made up of two expressions connected by an equal sign. Equation represents balance.	It may be a number, a variable, or a combination of numbers and variables and operation symbols.
With equal sign	Without equal sign
$3x = 2x + x$	$3x, 2x, x$



## ONE-VARIABLE EQUATIONS



**One-variable equation** is a mathematical statement that consists of a variable and whose left-hand side is exactly the same as what's on the right-hand side.

Examples of One-Variable Equation	Non-examples of One-Variable Equations
$2x = 4$	$2x - 3r$
$-m = 2$	$11c - 5$
$\frac{1}{2}r - 13 = 5$	$25m - 4n + 7$
$5(w - 7) = 26$	$100$



## PROPERTIES OF EQUALITY

Here are the commonly used properties of equality when solving one-variable equations. If  $a$ ,  $b$ , and  $c$  are real numbers, then...

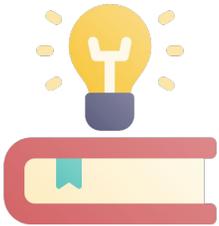
<b>Reflexive Property</b>	<b>Symmetric Property</b>	<b>Transitive Property</b>
$a = a$	If $a = b$ , then $b = a$	If $a = b$ and $b = c$ , then $a = c$
<b>Distributive Property</b>	<b>Addition Property of Equality (APE)</b>	<b>Multiplication Property of Equality (MPE)</b>
$a(b+c) = ab + ac$	If $a = b$ , then $a + c = b + c$	If $a = b$ , then $a \times c = b \times c$



# SOLVING ONE-VARIABLE EQUATIONS

Please take note of following steps in solving one-variable equations.

1. Simplify both sides of the equation.
2. Use the addition property of equality to put the terms with variable on one side of the equation and the constant terms on the other.
3. Use the multiplication property of equality to make the coefficient of the variable term equal to 1.
4. Check your answer by substituting your solution into the original equation.



**For example...**

For what value of  $x$  will make  $x - 5 = 8$  true?

Since both sides of the equation are already simplified, let's proceed to the next step.

$$x - 5 = 8$$

Use the addition property of equality to put the terms with variable on one side of the equation and the constant terms on the other.

$$x - 5 + 5 = 8 + 5$$

*We add 5 to both sides of the equation to isolate  $x$ . We do that to maintain the balance of the both sides of the equation.*



## SOLVING ONE-VARIABLE EQUATIONS

$$x - 5 + 5 = 8 + 5$$



$$x = 13$$

We skip the 3rd step because the coefficient of  $x$  is already 1. Thus, the value of  $x = 13$ .



So now that we get  $x$  as equal to 13, what does this mean?

Since  $x = 13$ , let us substitute 13 to  $x$  in the original equation. Thus,

$$\begin{aligned}x - 5 &= 8 \\(13) - 5 &= 8 \\13 - 5 &= 8 \\8 &= 8\end{aligned}$$



### Another example...

What is the value of  $m$  in  $3m + 8 = 1 + 2m$ ?



$$3m + 8 = 1 + 2m$$

*(Original Equation)*

$$3m - 2m + 8 = 1 + 2m - 2m$$

*(Add  $-2m$  on both sides, APE)*

$$m + 8 = 1$$

*(Simplified form)*

$$m + 8 - 8 = 1 - 8$$

*(Add  $-8$  on both sides, APE)*

$$m = -7$$

*(Simplified form)*

*Checking:* Substitute  $-7$  to  $m$ .

$$3(-7) + 8 = 1 + 2(-7)$$

$$-21 + 8 = 1 - 14$$

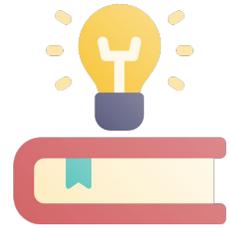
$$-13 = -13$$



## SOLVING ONE-VARIABLE EQUATIONS

### Another example...

What is the value of  $y$  in  $5(y + 2) = 20$ ?



$$5(y + 2) = 20$$

*(Original Equation)*

$$5y + 10 = 20$$

*(Distributive Property)*

$$5y + 10 - 10 = 20 - 10$$

*(Add -10 on both sides, APE)*

$$5y = 10$$

*(Simplified form)*

$$\left(\frac{1}{5}\right) 5y = 10 \left(\frac{1}{5}\right)$$

*(Multiply both sides by  $\frac{1}{5}$ , MPE)*

$$y = 2$$

*(Simplified form)*

*Checking:* Substitute 2 to  $y$ .

$$5(y + 2) = 20$$

$$5(2 + 2) = 20$$

$$5(4) = 20$$

$$20 = 20$$



## SOLVING ONE-VARIABLE EQUATIONS WITH FRACTION OR DECIMAL

When dealing with equation involving fractions or decimals, you may clear the fractions or decimals first in order to produce a simpler equation involving whole numbers.

1. To clear fractions, multiply both sides of the equation (distributing to all terms) by the LCD of all the fractions.
2. To clear decimals, multiply both sides of the equation (distributing to all terms) by the lowest power of 10 that will make all decimals whole numbers.



## SOLVING ONE-VARIABLE EQUATIONS WITH FRACTION OR DECIMAL

### Another example...

Solve for  $c$ :  $0.05c + 0.25 = 0.2$



$$0.05c + 0.25 = 0.2$$

*(Original Equation)*

$$100(0.05c + 0.25) = 0.2(100)$$

*(Multiply both sides by 100, MPE)*

$$5c + 25 = 20$$

*(Distributive Property)*

$$5c + 25 - 25 = 20 - 25$$

*(APE)*

$$5c = -5$$

*(Simplified form)*

$$\left(\frac{1}{5}\right) 5c = -5 \left(\frac{1}{5}\right)$$

*(Multiply both sides by  $\frac{1}{5}$ , MPE)*

$$c = -1$$

*(Simplified form)*

*Checking:* Substitute  $-1$  to  $c$ .

$$0.05c + 0.25 = 0.2$$

$$0.05(-1) + 0.25 = 0.2$$

$$-0.05 + 0.25 = 0.2$$

$$0.20 = 0.2$$

$$\mathbf{0.2 = 0.2}$$



## WORD PROBLEMS INVOLVING ONE-VARIABLE EQUATIONS



Chris has  $x$  dollars. After spending \$38 for his many school works and school supplies, he will have \$43 on his purse. What is the value of  $x$ ?



## WORD PROBLEMS INVOLVING ONE-VARIABLE EQUATIONS



Solution:

Let  $x$  = amount in dollars. Thus,

$$x - 38 = 43 \quad (\text{Rewrite the given as an equation})$$

$$x - 38 + 38 = 43 + 38 \quad (\text{APE})$$

$$x = 81 \quad (\text{Simplified form})$$

**Chris has \$ 81.**

## PRACTICE EXERCISES



Directions: Solve for the unknown variable. Show your complete solution.

1.  $x - 10 = 4$

2.  $4c - 1 = 8$

3.  $\frac{1}{2}x = 5 + x$

4.  $-2(a - 3) = 12$

5.  $0.25m = 50$

6.  $2r + 1 = 3r - 17$



# TABLE OF ACTIVITIES

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# THE GRADUATION SPEECH

Let's now listen to the graduation speech. The speaker pointed out one-variable equations. Identify which among the following are equations or expressions. If it is only an expression, state the reason why.

1.  $a + b = c$

2.  $3r$

3.  $2(m+n) + 5$

4.  $8p - 7 = 1$

5.  $13q - 6 + 5$

6.  $\frac{1}{3} + y = 2$

7.  $11.3x - 12p$

8.  $3r - 15 + y(x)$

9.  $x + 1$

10.  $3y = 5$

11.  $3x - 2 + 5$

12.  $y = 0.05$

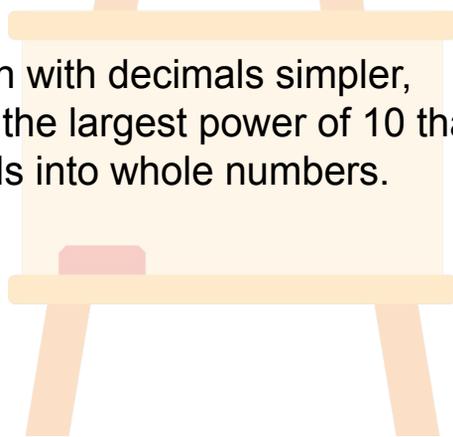
13.  $23x + 3.5y$



## THE PROFESSOR'S REMINDER

Do you still remember the professor's reminder about one-variable equations yesterday? Determine if the following statements are TRUE or FALSE. Write TRUE if it's correct, otherwise replace the underlined word/phrase to make the statement true.

- \_\_\_\_\_ 1. An equation is an algebraic expression with equal sign.
- \_\_\_\_\_ 2. Expressions are consist of a variable and whose left-hand side is the same as the right-hand side.
- \_\_\_\_\_ 3.  $-a + 2 = 5$  is an example of equation in one variable.
- \_\_\_\_\_ 4. Given that  $x$  is a real number,  $x = x$  because of transitive property.
- \_\_\_\_\_ 5. APE states that what is added on the left-hand side should also be added to the right-hand side.
- \_\_\_\_\_ 6. To clear out fractions in an equation, multiply each term by the GCF of all the fractions.
- \_\_\_\_\_ 7. To make an equation with decimals simpler, divide both sides by the largest power of 10 that will make all decimals into whole numbers.



## WRITING DOWN NOTES

Make sure to complete all your notes in preparation for the upcoming examination! As practice, write an equation that will represent the given situation below.



1. The school is  $x$  miles away from your home. You have travelled back and forth and covered a distance of 46 miles. Find  $x$ .



2. Brent has  $m$  new notebooks. If he adds 3 more, he will have 9 notebooks. Find  $m$ .

3. Clark bought a new study desk that costs \$ 119. He paid \$  $4z$  and got a change of \$ 6. What is the value of  $z$ ?

4. You have studied for 120 minutes. 34 minutes was used for math subject, 58 for science subject, and  $w$  minutes for English. Find  $w$ .



## BALANCING SCHOOL ERRANDS

Balancing school works makes our study habit more productive. In line with this, can you also look for the equivalent expressions of the following? The first one is done for you.

Example: What is the equivalent expression of

$$4(n + 1) + 2(n - 5)?$$

Solution:

$$4(n + 1) + 2(n - 5) = 4n + 4 + 2n - 10$$

$$= 4n + 2n + 4 - 10$$

$$= 6n - 6$$

**Thus,  $4(n + 1) + 2(n - 5) = 6n - 6$**



1. $y + y + y - 5$	2. $2(x + 8) + 3x$	3. $0.2p - 4(p - 5)$
4. $2x + x - 5x - 7$	5. $20z + 45 - 30$	6. $\frac{1}{5}x + \frac{3}{4}x + 20$



## SEARCHING FOR ANSWERS

Like in school, teachers ask questions and you are required to answer. How about the following questions below? Solve for the value of the variables.

1.  $4c = 4$

Solution:

2.  $6 + b = 7$

Solution:

3.  $\frac{1}{3}s = -9$

Solution:

4.  $2w + 4 = 8$

Solution:

5.  $14 = 3 + 3r$

Solution:



## TARGET: A +

It's quiz time! Our goal is to have an A+ grade! Find the value of each variable. Don't forget to show your complete solution.

1.  $3(a - 2) = 16$

2.  $-3(2k - 8) = -12$

3.  $4(6+2z) = 0$

4.  $3a + 2a + 6 = -15$

5.  $4 = -2(d + 3)$

6.  $27 = 46 + 2c - c$

7.  $4(2t - 3) + 4 = 8t - 8$

8.  $6h + 11 = -(6h + 5)$

9.  $9c - 6 = -3c + 30$



# CHALLENGING EXAMINATION

Let's raise the bar of difficulty. Solve for the unknown value.

1.  $4(x + 3) - 3(x + 1) = 5$

2.  $-5(3 - 4b) = -6 + 20b - 9$

3.  $\frac{n}{10} = 9 - \frac{n}{5}$

4.  $28(a + 2) - 7(3a + 11) = 1$

5.  $\frac{9}{4} = \frac{-20}{5}$

6.  $1.05x + 3.21x = 54.19$



## MR. ACE: WHAT WENT WRONG?

Mr. Ace is checking the quiz of his class. When apparently, he notice that there is something wrong with the answer of his student. Help Mr. Ace point out the errors.

$$10m + 5 = 8 + 5m$$

$$10m - 5m + 5 = 8 + 5m$$

$$5m + 5 = 8 + 5m$$

$$5m + 5m = 8 - 5$$

$$10m = 3$$

$$m = 3.33$$

*(Original Equation)*

*(APE)*

*(Simplified form)*

*(APE)*

*(Simplified form)*

*(MPE, Simplified form)*



What went wrong with the solution and the answer? Write it down.

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Provide the new solution with checking.



# HOMEWORK: FACT CHECK

Look at your classmates' answers on homework! Are they really correct? Write YES if it is, otherwise provide the correct answer.



1.  $2(p + 4) = -3(p + 5)$   
Answer is  $p = 12$

2.  $3r = 15$   
Answer is  $r = 5$ .

3.  $\frac{1}{3}x = 4$   
Answer is 16

4.  $9w - 6 = -3w + 30$   
Answer is  $w = 3$ .



# THE SCHOOL LIFE

These are common situations in school. Solve the following.

1. Stephanie, Mae, and Maria took a long quiz yesterday. This morning, they have received the result. The total of 3 scores is 63. Their individual scores are:

Stephanie:  $x$

Mae:  $x + 3$

Maria :  $2x$

Find the value of  $x$ , and their individual scores.



2. Stephanie, Mae, and Maria took another quiz. This time, the total of their scores is 103. Their individual scores are:

Stephanie:  $x$

Mae:  $x + 10$

Maria :  $x + 18$

Find the value of  $x$ , and their individual scores.



# ANSWER GUIDE

## Activity 1

1. Equation
2. Expression, just a number and variable
3. Expression, no equal sign
4. Equation
5. Expression, no equal sign
6. Equation
7. Expression, no equal sign
8. Expression, no equal sign
9. Expression, no equal sign
10. Equation
11. Expression, no equal sign
12. Equation
13. Expression, no equal sign

## Activity 2

- |              |                           |             |
|--------------|---------------------------|-------------|
| 1. True      | 2. One- variable equation | 3. True     |
| 4. Reflexive | 5. True                   | 6. LCD      |
|              |                           | 7. Multiply |

## Activity 3

- |              |              |                 |                 |
|--------------|--------------|-----------------|-----------------|
| 1. $2x = 46$ | 2. $m+3 = 9$ | 3. $4z-6 = 119$ | 4. $92+w = 120$ |
|--------------|--------------|-----------------|-----------------|

## Activity 4

- |              |               |               |
|--------------|---------------|---------------|
| 1. $3y-5$    | 2. $5x+16$    | 3. $-3.8p+20$ |
| 4. $-2x - 7$ | 5. $20z + 15$ | 6. $9x + 400$ |



# ANSWER GUIDE

## Activity 5

1.  $c=1$       2.  $b=1$       3.  $s=-27$       4.  $w=2$       5.  $r=11/3$

## Activity 6

1.  $a=22/3$       2.  $k=6$       3.  $z=-3$       4.  $a=-21/5$       5.  $d=-5$   
6.  $c=-19$       7. no solution      8.  $h=4/3$       9.  $c=3$

## Activity 7

1.  $x=-4$       2. No solution      3.  $n=30$   
4.  $a=22/7$       5.  $g=-16$       6.  $x=5419/426$

## Activity 8

The correct answer is  $m = \frac{3}{5}$ . The solution did not perform APE properly.  $-5m$  must be added to both sides of the equation.

## Activity 9

1.  $P= -23/5$       2. Yes      3.  $x=12$       4.  $w=3$

## Activity 10

1.  $X = 15$ , Stephanie = 15, Mae = 18, Maria = 30  
2.  $X = 25$ , Stephanie = 25, Mae = 35, Maria = 43



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