# Helping With Math $\begin{gathered}\text { UsA } \\ \text { cwes } \\ \text { den }\end{gathered}$ 

## Proper and Improper Fractions

## Suitable for students aged 8-10

## Proper Fraction

Proper fraction is a type of fraction whose numerator is always less than the denominator. In symbol,

Numerator < Denominator

$$
\begin{aligned}
& \frac{2}{3} \text { Numerator } \\
& \text { Denominator }
\end{aligned}
$$

## Improper Fraction

Improper fraction is a type of fraction whose numerator is always greater than or equal to the denominator. In symbol,
Numerator $\geq$ Denominator
$3 \longrightarrow$ Numerator
$2 \longrightarrow$ Denominator

Examples of proper fractions

three-fifths

two-thirds

seven-eights

## PROPER AND IMPROPER FRACTIONS

Examples of improper fractions

three-thirds

eight-fifths

## ADDITION OF PROPER AND IMPROPER FRACTIONS

## STEPS IN ADDING PROPER AND IMPROPER FRACTIONS

§tep 1: Make sure that the denominators are the same. If not,
express the fractions as like fractions.
Step 2: Add the top numbers (the numerators), put that
answer over the denominator.
Step 3: Simplify the fraction (if needed).

## ILLUSTRATIVE EXAMPLES

Solve for the sum of the following fractions:

1. $\frac{1}{4}+\frac{2}{4}$
2. $\frac{5}{4}+\frac{4}{3}$
3. $\frac{1}{3}+\frac{7}{3}+\frac{4}{6}$

## ADDING PROPER AND IMPROPER FRACTIONS

## SOLUTIONS:

1. $\frac{1}{4}+\frac{2}{4} \Rightarrow \frac{1+2}{4}=\frac{3}{4}$
2. $\frac{5}{4}+\frac{4}{3}$

Rewrite the given as like fractions. The LCD is 12 .

$$
\begin{aligned}
& \Rightarrow \frac{5}{4}=\frac{(12 \div 4) \times 5}{12}=\frac{3 \times 5}{12}=\frac{15}{12} \\
& \Rightarrow \frac{4}{3}=\frac{(12 \div 3) \times 4}{12}=\frac{4 \times 4}{12}=\frac{16}{12} \\
\frac{5}{4}+\frac{4}{3} & \Rightarrow \frac{15}{12}+\frac{16}{12}=\frac{15+16}{12}=\frac{31}{12} \text { or } 2 \frac{7}{12}
\end{aligned}
$$

3. $\frac{1}{3}+\frac{7}{3}+\frac{4}{6} \Rightarrow$ Rewrite the given as like fractions. The LCD is 6 .

$$
\begin{aligned}
& \Rightarrow \frac{1}{3}=\frac{(6 \div 3) \times 1}{6}=\frac{2 \times 1}{6}=\frac{2}{6} \\
& \Rightarrow \frac{7}{3}=\frac{(6 \div 3) \times 7}{6}=\frac{2 \times 7}{6}=\frac{14}{6} \\
& \Rightarrow \frac{4}{6}=\frac{(6 \div 6) \times 4}{6}=\frac{1 \times 4}{6}=\frac{4}{6}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{3}+\frac{3}{7}+\frac{4}{6} & \Rightarrow \frac{2}{6}+\frac{14}{6}+\frac{4}{6}=\frac{2+14+4}{6} \\
& =\frac{2+14+4}{6}=\frac{20}{6} \text { or } 3 \frac{2}{6} \text { or } 3 \frac{1}{3}
\end{aligned}
$$

## SUBTRACTION OF PROPER AND IMPROPER FRACTIONS

## STEPS IN SUBTRACTING PROPER AND IMPROPER FRACTIONS

Step 1: Make sure that the denominators are the same. If not, express the fractions as like fractions.

Step 2: Subtract the top numbers (the numerators), put that ! answer over the denominator.
$\star$ Step 3: Simplify the fraction (if needed).

## ILLUSTRATIVE EXAMPLES

Solve for the difference of the following fractions:

1. $\frac{3}{4}-\frac{1}{4}$
2. $\frac{7}{4}-\frac{4}{3}$

SOLUTIONS:

1. $\frac{3}{4}-\frac{1}{4} \Rightarrow \frac{3-1}{4}=\frac{2}{4}=\frac{1}{2}$

2. $\frac{7}{4}-\frac{4}{3}$

$$
\begin{aligned}
& \Rightarrow \frac{7}{4}=\frac{(12 \div 4) \times 7}{12}=\frac{3 \times 7}{12}=\frac{21}{12} \\
& \Rightarrow \frac{4}{3}=\frac{(12 \div 3) \times 4}{12}=\frac{4 \times 4}{12}=\frac{16}{12} \\
\frac{5}{4}-\frac{4}{3} & \Rightarrow \frac{21}{12}-\frac{16}{12}=\frac{21-16}{12}=\frac{5}{12}
\end{aligned}
$$

## MULTIPLICATION OF PROPER AND IMPROPER FRACTIONS



To solve the problem, we need to multiply the two proper fractions. Look at the illustration below.


Illustrate the given fractions by using a model. Combine the two models and look at the overlapping area. Then, simplify your answer if needed.

## MULTIPLICATION OF PROPER AND IMPROPER FRACTIONS

To multiply fractions, simply multiply the numerators and the denominators. Then, write the product in its simplest form.


## $3 \div 3$

$12 \div 3$

## You can simplify the product because both 3 and 12 can still be divided by 3 .



Lowest term

Canceling may be used to eliminate common factors before multiplying the numerators and denominators.


## DIVISION OF PROPER AND IMPROPER FRACTIONS



STEP 1 Identify the first fraction (dividend) and the second fraction (divisor)


STEP 2 Reciprocate the second fraction or interchange the numerator and the denominator.

$$
\uparrow \frac{1}{3} \downarrow \quad \sum \ggg \frac{3}{1}
$$

STEP 3 Multiply the first fraction to the reciprocated second fraction.

$$
\frac{5}{6} \times \frac{3}{1}=\frac{15}{6}
$$

STEP 3
Simplify if necessary.

$$
\frac{15 \div 3}{6 \div 3}=\frac{5}{2}
$$

## EXAMPLES:

1.)

$$
\frac{1}{9} \div \frac{2}{5}
$$

2.)
$\frac{7}{8} \div \frac{1}{2}$

## SOLUTIONS:

1.)

$$
\begin{aligned}
& \frac{1}{9} \div \frac{2}{5} \\
= & \frac{1}{9} \times \frac{5}{2} \\
= & \frac{5}{18}
\end{aligned}
$$

3.)

$$
\begin{aligned}
& \frac{3}{2} \div \frac{9}{8} \\
= & \frac{3}{2} \times \frac{8}{9} \\
= & \frac{24}{18}=1 \frac{1}{3}
\end{aligned}
$$

3.)
$\frac{3}{2} \div \frac{9}{8}$
4.)
$\frac{7}{2} \div \frac{9}{4}$
2.)

$$
\begin{aligned}
& \frac{7}{8} \div \frac{1}{2} \\
= & \frac{7}{8} \times \frac{2}{1} \\
= & \frac{14}{8}=\frac{7}{4}
\end{aligned}
$$

4.)

$$
\begin{aligned}
& \frac{7}{2} \div \frac{9}{4} \\
= & \frac{7}{2} \times \frac{4}{9} \\
= & \frac{28}{18}=1 \frac{5}{9}
\end{aligned}
$$

## TABLE OF ACTIVITIES

| Ages 8-9 (Basic) |  |
| :---: | :--- |
| 1 | The Snow |
| 2 | Sth Grade |
| 3 | Snowman Cream |
| 4 | A Cup of Hot Chocolate |
| 5 | Sweater |
|  | Ages 9-10 (Advanoed) |
| 6 | Warm Bubble Bath |
| 7 | Winter Camping |
| 8 | Fireplace |
| 9 | One Very Cold Night |
| 10 | Ice Skate on a Lake |

## THE SNOW

It's winter season! Jones' family are eating lunch while seeing the view outside their house and they saw blocks of snow. The shaded portion below represents the space covered by the snow. Identify the fraction being illustrated.

2.

4.

5.


After taking a bath, Mrs. Jones wants to make a snow cream that will best fit with their hot brewed coffee. She asks her children to help her and add these measurements to make a perfect snow cream.

1. $\frac{2}{5}+\frac{3}{5}$
2. $\frac{7}{9}+\frac{5}{6}$

$$
\begin{aligned}
& \text { 3. } \frac{4}{7}+\frac{7}{8}
\end{aligned}
$$

## SNOWMAN

Mr. and Mrs. Jones' children want to build a snowman in their veranda. They decided to play a game, whoever answers the question correctly will have the chance to make a step in building the snowman, whoever builds the snowman first will be the winner.
1.) Find the difference of the fractions.
$\frac{5}{9}-\frac{1}{4}$
2.)
$\frac{6}{7}-\frac{2}{3}$
3.)
$\frac{3}{5}-\frac{3}{10}$
4.)

$$
\frac{5}{8}-\frac{7}{12}
$$

## SOLUTIONS:



## A CUP OF HOT CHOCOLATE

It's freezing outside, and Jones' family would like to drink some chocolate drink. Alice wants to have two cups of chocolate drink, but she needs to answer first her father's questions about the multiplication of fractions.

Multiply the fractions.

1. $\frac{2}{3} \times \frac{1}{2}$
2. $\frac{3}{7} \times \frac{4}{5}$
3. $\frac{6}{10} \times \frac{2}{9}$
4. $\frac{7}{10} \times \frac{5}{14}$

## SWEATER

Since its winter season, Jones' family needs to wear layers of sweaters. They went to the mall to buy one extra sweater for each family member. But, whoever answers the operation correctly will get the corresponding sweater.

1. $\frac{5}{9} \div \frac{2}{5}$
2. $\frac{1}{2} \div \frac{7}{9}$
3. $\frac{2}{3} \div \frac{10}{13}$

## WARM BUBBLE BATH

Jones' family would like to take a warm bubble bath to lessen the cold they feel. Mr. Jones administers a game, whoever the winner will have a longer time taking a bath.

Identify the fraction being illustrated.

3.



## WINTER CAMPING

Mr. Jones and her brother decided to have winter camping. They arranged the things they would bring, but they needed to have a limited number of kilograms in total to walk a mile. Help them to weigh their belongings.


1. $\frac{5}{2}+\frac{6}{5}$
2. $\frac{7}{3}+\frac{7}{4}$

3. $\frac{11}{7}+\frac{8}{5}$
4. $\frac{13}{6}+\frac{3}{2}$

## FIREPLACE

Mr. Jones bought some firewood for their fireplace. The measurement of each firewood in meters will be lessened by a certain length because it doesn't fit the fireplace. Help Mr. Jones to get the answer.
1.) $\frac{3}{2}-\frac{5}{4}$
2.) $\frac{9}{2}-\frac{6}{5}$

3.) $\frac{13}{4}-\frac{11}{9}$
4.) $\frac{8}{3}-\frac{9}{8}-\frac{5}{4}$

## ONE VERY COLD NIGHT

Mrs. Jones would like to make a soup that she watched on Youtube for a freezing evening. The measurement of every ingredient she needs is the answer to each equation below. Help Mrs. Jones to find the exact measurement.

$$
\text { 1.) } \frac{4}{3} \times \frac{5}{3}
$$

2.) $\frac{7}{5} \times \frac{20}{14}$
3.) $\frac{9}{5} \times \frac{25}{18}$

$$
\text { 4.) } \frac{9}{8} \times \frac{14}{10} \times \frac{36}{27}
$$

## ICE SKATE ON A LAKE

Marcus, son of Mr. Jones, wants to go ice skating on a lake with his friends. He needs to help her younger brother first before he can go by dividing the fractions below.

1. $\frac{10}{3} \div \frac{5}{2}$
2. $\frac{11}{4} \div \frac{9}{5}$
3. $\frac{6}{5} \div \frac{10}{9}$
4. $\frac{12}{5} \div \frac{9}{7} \div \frac{14}{10}$

## ANSWER GUIDE

## Activity 1

1.) $\frac{4}{7}$
2.) $\frac{3}{5}$
3.) $\frac{8}{9}$
4.) $\frac{1}{12}$
5.) $\frac{3}{4}$

Activity 2
1.) 1
2.) $1 \frac{11}{18}$
3.) $1 \frac{21}{56}$
4.) $2 \frac{1}{18}$

Activity 3
1.) $\frac{11}{36}$
2.) $\frac{4}{21}$
3.) $\frac{3}{10}$
4.) $\frac{1}{24}$

Activity 4
1.) $\frac{1}{3}$
2.) $\frac{12}{35}$
3.) $\frac{2}{15}$
4.) $\frac{1}{2}$

Activity 5
1.) $1 \frac{17}{18}$
2.) $\frac{9}{14}$
3.) $\frac{13}{15}$
4.) $\frac{2}{3}$

## ANSWER GUIDE

## Activity 6

1.) $\frac{4}{4}=1$
2.) $\frac{5}{3}$
3.) $\frac{17}{12}$
4.) $\frac{7}{7}=1$

Activity 7
1.) $3 \frac{7}{10}$
2.) $4 \frac{1}{12}$
3.) $3 \frac{6}{35}$
4.) $3 \frac{2}{3}$

Activity 8
1.) $\frac{1}{4}$
2.) $3 \frac{3}{10}$
3.) $2 \frac{1}{36}$
4.) $\frac{7}{24}$

Activity 9
1.) $2 \frac{2}{9}$
2.) 2
3.) $2 \frac{1}{2}$
4.) $2 \frac{1}{10}$

Activity 10
1.) $1 \frac{1}{3}$
2.) $1 \frac{19}{36}$
3.) $1 \frac{2}{25}$
4.) $1 \frac{1}{3}$

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